

St John's Gosport C of E Primary School

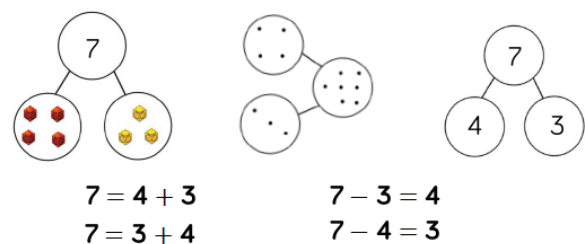


Progression in Calculation and Number Facts

KS1 and KS2

November 2024

Part-whole



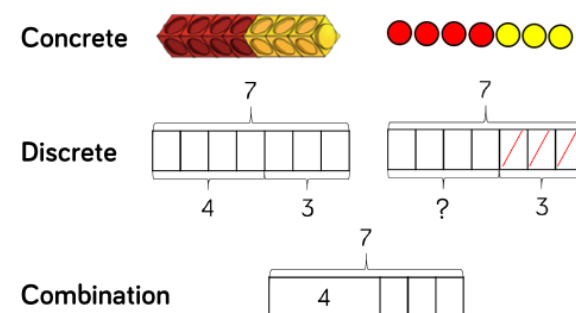
Supports understanding of partitioning and aggregation. When the whole part is incomplete, the children can use aggregation to add the other 2 parts together to find a total.

When one part and the whole is known, the children can use subtraction to find the missing part.

Part-whole models can be used to partition numbers in a variety of ways including into their place value columns (hundred, tens, ones etc.) also to partition other ways which can make the whole, this is known as flexible partitioning (e.g. $430 = 1$ hundred 33 tens).

This model also supports understanding of larger numbers in UKS2 and decimals, fractions and percentages.

Bar Models (single)

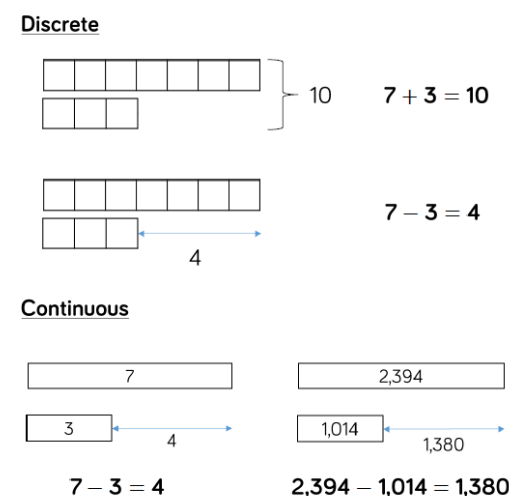


This supports children in representing the structure of a calculation (similar to the part-whole model).

Concrete resources, such as counters and cubes, can be used alongside bar models to show the connection between the concrete and the pictorial.

They can be used to represent larger numbers, fractions, decimals and percentages.

Bar Model (multiple)

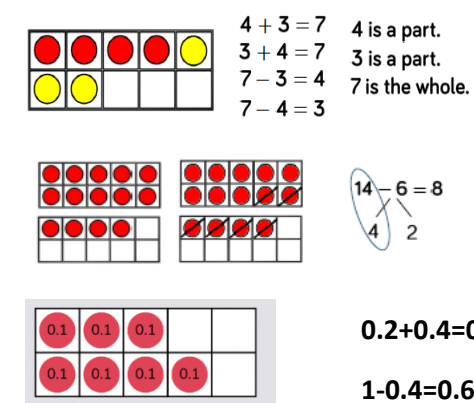


The multiple bar model is useful when comparing quantities whilst looking at the structure of the numbers and calculations.

Two or more bars can be used and the whole is represented with the use of a bracket at the side

Smaller number work better using a discrete bar model whereas larger numbers should be represented on continuous bar models.

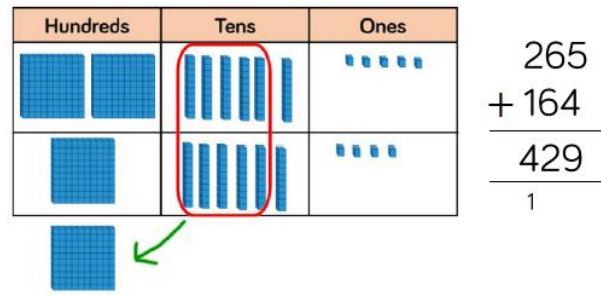
Tens Frame



When adding 2 single digit numbers, tens frames support children's understanding of the structures of addition and subtraction. This further develops their knowledge of aggregation where parts combined make a whole and therefore the whole is split into parts. Tens frames also develop understanding of augmentation (increasing a number) and taking away (decreasing a number).

Tens frames can be developed as numbers increase through the use of one part of the tens frame representing a different number. For example, showing that 10 tens make 100 and similarly can be applied to their understanding of decimal numbers.

Dienes/Base 10 (addition)



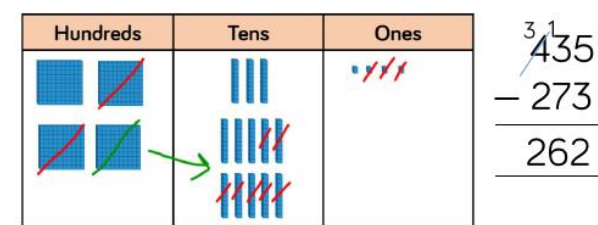
This is one of the most effective concrete resource to support children's understanding of column addition. These should be used alongside the children's written calculations.

First children should add without exchanging then move on to exchanging with one column and then one or more columns.

This is a less effective choice for larger numbers due to the size of the dienes, in which case Place Value counters are the most effective manipulative.

Adding must always begin the smallest place value column.

Dienes/Base 10 (subtraction)

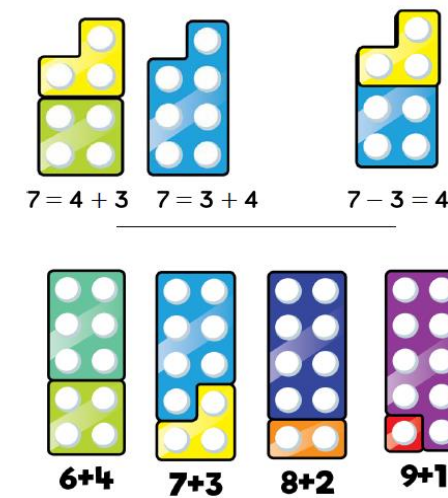


This is an effective concrete resource to support children's understanding of column subtraction. These should be used alongside the children's written calculations.

Clear links should be provided between the concrete and the abstract representations. The abstract should be in the form of expanded methods in LKS2.

This is must efficient with up to 4-digit numbers, after which place value counters should be used.

Numicon

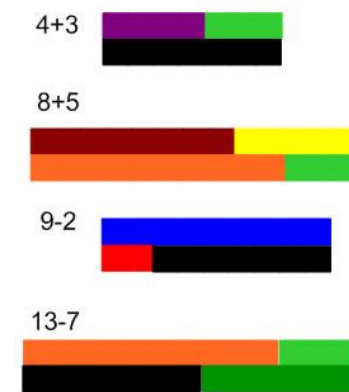


Numicon can be used to support children subitise numbers as well as develop an understanding of aggregation, partitioning and number bonds.

When adding children can develop their conceptual understanding of parts making a whole as well as using parts on top of the whole to understand subtraction.

It can be used a systematic tool to develop knowledge of number bonds.

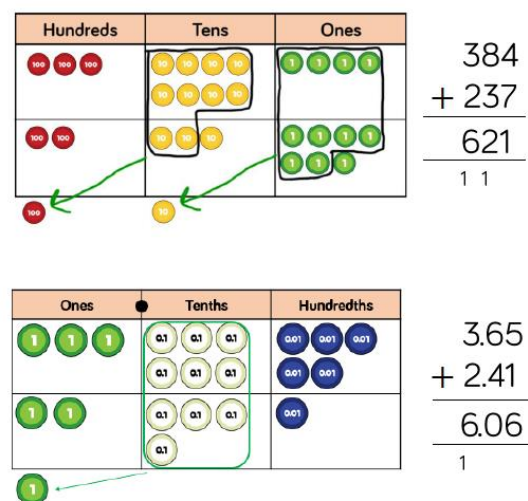
Cuisenaire



Cuisenaire is a great tool to use alongside number lines and bar models. The value of the rod can be determined by the calculation and does not always have to represent a single digit number.

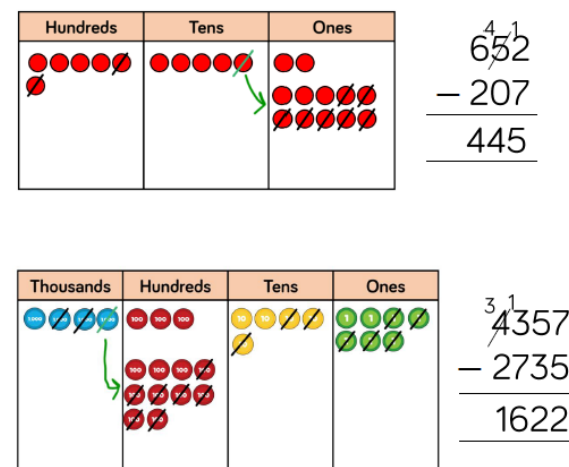
It can be used systematically, like Numicon, to discover the patterns in number bond.

Place Value Counters (addition)



Using place value counters is an effective way to support the children's understanding of column addition. It should be used alongside the children's written calculation (expanded, compact column addition).

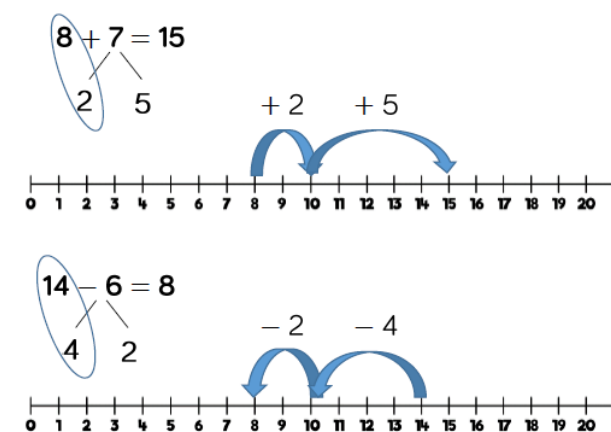
First they should add without exchanging before moving on to additions with exchanges. This is suitable for larger whole numbers as well as decimal representations of addition.



Using place value counters is an effective way to support the children's understanding of column subtraction. It should be used alongside the children's written calculation (expanded, compact column subtraction).

First they should subtract without exchanging before moving on to subtractions with exchanges. This is suitable for larger whole numbers as well as decimal representations of subtraction.

Number Lines (structured)

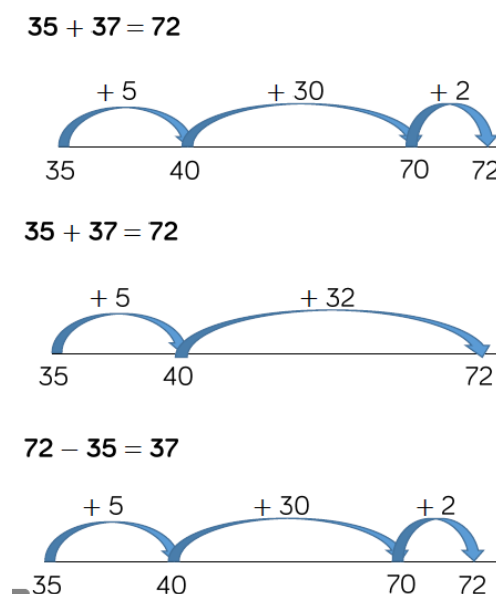


Structured number lines support children's understanding of addition and subtraction augmentation and reduction.

It also allows children to develop their understanding of the value of numbers and their use of flexible partitioning to make jumps on the number line that are more effective. For example, jumping on or back to the nearest 10.

These can be used alongside concrete resources such as dienes and cubes.

Number Line (unstructured)



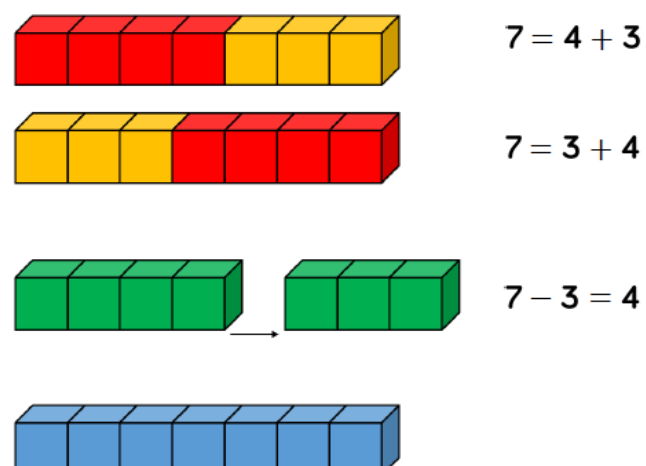
Blank (unstructured) number lines allow children to add and subtract in manageable "chunks".

When subtracting, it gives children the opportunity to explore the concept of the difference between two numbers by either counting on or counting back and developing their understanding of the amount between them.

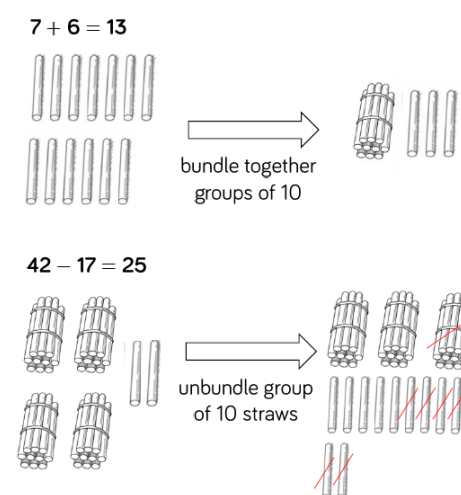
Unstructured number lines should be used when calculating differences in time.

Other Resources (concrete and pictorial)

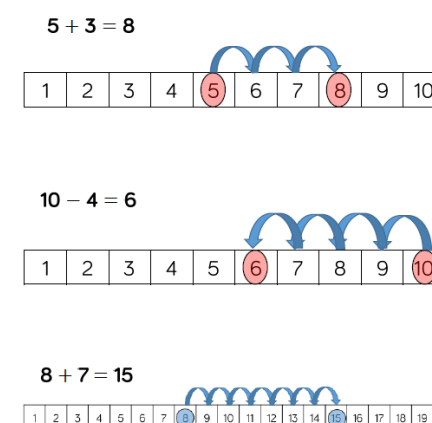
Cubes/Multi-link



Straw bundles



Number Tracks



Progression of Skills Addition and Subtraction Overview: Years 1-6

Addition			
Skill	Year Group	Representations and Models	
Add 1 and 2 digit numbers to 20	1	Part-whole Bar Model Bead strings Structured, labelled number lines	Numicon Tens frame (within 20) Number tracks
Add three 1 digit numbers	2	Part-whole Bar model	Ten frames (within 20) Numicon
Add 1 and 2 digit number to 100	2	Part-whole Bar model Straws	Number lines (structured and unstructured) Hundred squares
Add two 2 digit numbers	2	Part-whole Bar model Number lines (unstructured)	Straws Dienes/Base 10 Place Value Counters
Add with up to 3 digits	3	Part-whole Bar model Dienes/Base 10	Place Value Counters Column Addition
Add with up to 4 digits	4	Part-whole Bar model Dienes/Base 10	Place Value Counters Column Addition
Add with more than 4 digits	5/6	Part-whole Bar model	Place Value Counters Column Addition
Add with up to 3 decimal place	5/6	Part-whole Bar model	Place Value Counters Column Addition
Subtraction			
Skill	Year Group	Representations and Models	
Subtract two 1-digit numbers to 20	1	Part-whole model Bar model Numicon	Tens frames Bead strings Number tracks
Subtract 1 and 2 digit numbers to 100	2	Part-whole model Bar model Number lines (structured)	Number lines (unstructured) Straws Hundred Square
Subtract two 2-digit numbers	2	Part-whole model Bar model Number lines (unstructured)	Dienes/Base 10 Place Value Counters
Subtract with up to 3 digits	3	Part-whole model Bar model	Dienes/Base 10 Place Value Counters Column Addition
Subtract with up to 4 digits	4	Part-whole model Bar model	Dienes/Base 10 Place Value Counters Column Addition

Subtract with more than 4 digits	5/6	Part-whole model Bar model	Place Value Counters Column Addition
Subtract with up to 3 decimal places	5/6	Part-whole model Bar model	Place Value Counters Column Addition

Number Facts – Year 1			
Number and Place Value	Addition and Subtraction	Measure	Fractions
<ul style="list-style-type: none"> Know the sequence of counting in multiples of 2 Know the sequence of counting in multiples of 10 Know the sequence of counting in multiples of 5 Say one more or one less than any number up to 20. 	<ul style="list-style-type: none"> Know the number bonds and related subtraction facts for all numbers to 5 For example: <div> $4 + 0 = 4$ $4 - 0 = 4$ $3 + 1 = 4$ $4 - 1 = 3$ $2 + 2 = 4$ $4 - 2 = 2$ $1 + 3 = 4$ $4 - 3 = 1$ $0 + 4 = 4$ $4 - 4 = 0$ </div> Know the number bonds for all numbers to 10 and the related subtraction facts Know the number bonds for all numbers up to 20 and the related subtraction facts For example <div> $10 + 2 = 12$ $12 - 2 = 10$ $9 + 3 = 12$ $12 - 3 = 9$ $8 + 4 = 12$ $12 - 4 = 8$ </div> Recognise that ‘teens’ numbers comprise of one ten and some ones. 	<ul style="list-style-type: none"> Say the days of the week and the months of the year in the correct order Recognise the coins and notes of the realm and starting with 1p, 2p, 5p, 10p, 20p Apply number bond knowledge to coins 10p+1p= 11p 10p+2p=12p 	<ul style="list-style-type: none"> Know that: $\frac{1}{2} + \frac{1}{2} = 1$ whole $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$ whole

Models and images to support conceptual understanding of year 1 number facts

2p coins 10p coins 5p coins

Fourteen is one ten and four ones
 $14 = 10 + 4$

$3 + 1 = 4$
Tens frame with addition equation

	Blue	Red
Row 1	0	6
Row 2	1	5
Row 3	2	4
Row 4	3	3
Row 5	4	2
Row 6	5	1
Row 7	6	0

Systematic patterning to partition six

Number line to support counting in multiples of 2

$7 - 3 = 4$
Cherry partitioning model with subtraction equation

$2 + 3 = 5$
Number line with addition equation

Year 1: Addition

Linked to Hampshire Schemes of Learning Units: 1.1, 1.2, 1.4, 1.5, 1.7, 1.8, 1.9 and 1.12.

National Curriculum Objectives:

Pupils should be taught to:

- represent and use number bonds and related subtraction facts within 20.
- add and subtract 1-digit and 2-digit numbers to 20, including 0.

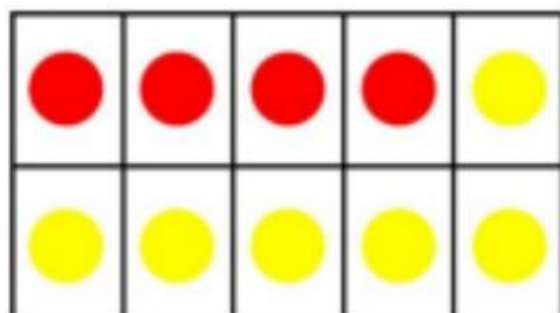
The Big Ideas:

Relating numbers to 5 and 10 helps develop knowledge of the number bonds within 20. For example, given $8 + 7$, thinking of 7 as $2 + 5$ and adding the 2 to 8 to make 10 and then the 5 to total 15. Thinking of part whole relationships is helpful in linking addition and subtraction. For example, where the whole is 6, and 4 and 2 are parts. This means that 4 and 2 together form the whole, which is 6 and 6 subtract 4 leaves the 2 and 6 subtract 2 leaves the 4.

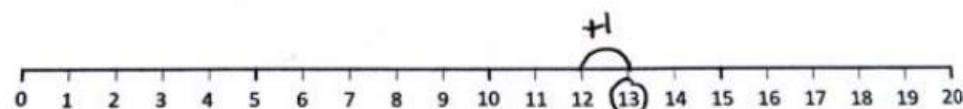
Stage 1

Represent number bonds within 10.

$$4 + 6 = 10$$



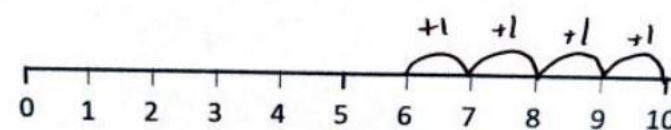
Find 1 more from any given number within 20.



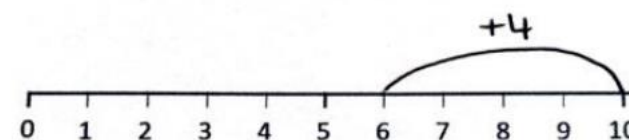
$$12 + 1 = 13$$

Stage 2

Use number bonds within 10.

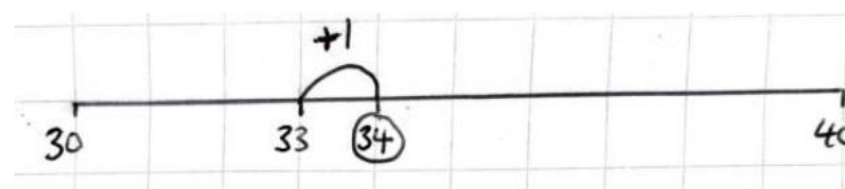


$$6 + 4 = 10$$



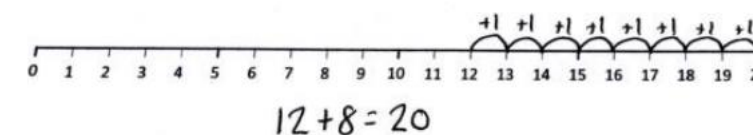
$$6 + 4 = 10$$

Find 1 less from any given number within 50.

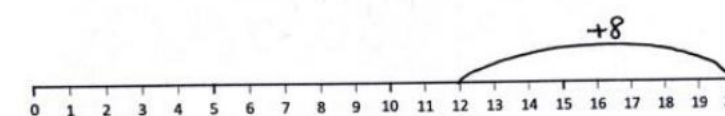


End of Year Expectation

Represent and use number bonds within 20.

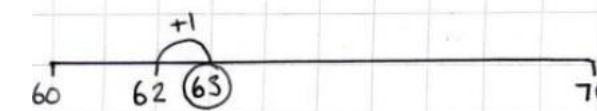


$$12 + 8 = 20$$

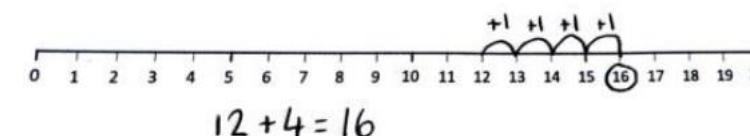


$$12 + 8 = 20$$

Find 1 more from any given number within 100.

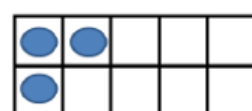


Add 1-digit and 2-digit numbers to 20.



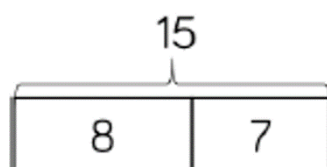
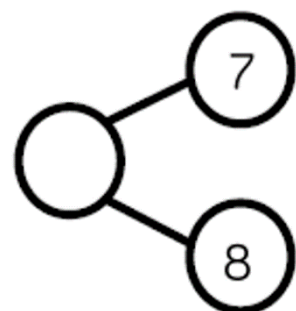
$$12 + 4 = 16$$

Year 1 Addition: Representations and Models



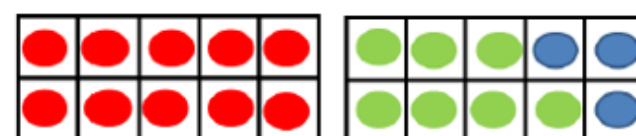
$$3 + \square = 10$$

How many to make 10? What about 20?



Partitioning numbers in different ways

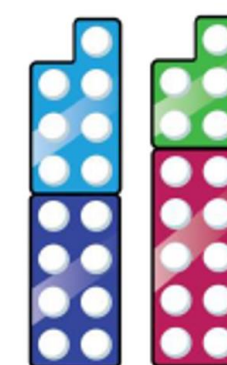
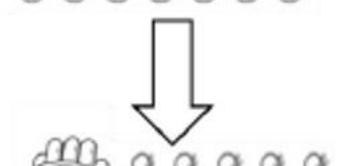
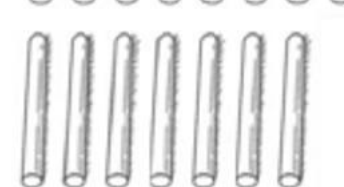
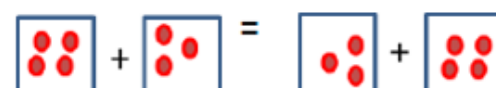
$$20 = 10 + 7 + \square$$



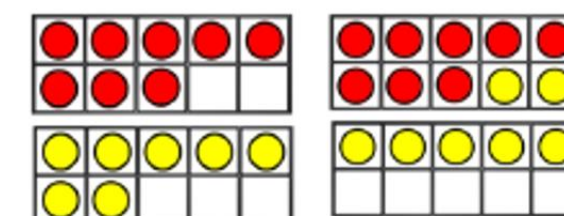
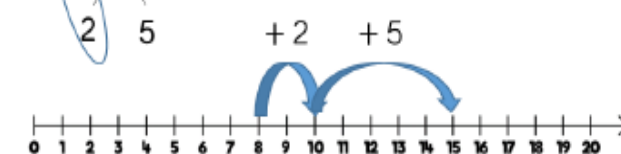
Lucy has 20 marbles in her bag. 10 were red, 7 were green and the rest were blue. How many

Commutativity

$$4 + 3 = 3 + 4$$



$$8 + 7 = 15$$



$$8 + 7 = 15$$



Year 1: Subtraction

Linked to Hampshire Scheme of Learning Units: 1.1, 1.2, 1.4, 1.5, 1.7, 1.8, 1.9 and 1.12.

National Curriculum Objectives:

Pupils should be taught to:

- represent and use number bonds and related subtraction facts within 20
- add and subtract 1-digit and 2-digit numbers to 20, including 0

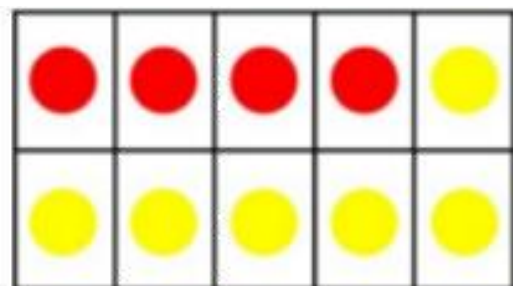
The Big Ideas:

Relating numbers to 5 and 10 helps develop knowledge of the number bonds within 20. For example, given $8 + 7$, thinking of 7 as $2 + 5$ and adding the 2 to 8 to make 10 and then the 5 to total 15. Thinking of part-whole relationships is helpful in linking addition and subtraction. For example, where the whole is 6, and 4 and 2 are parts. This means that 4 and 2 together form the whole, which is 6 and 6 subtract 4 leaves the 2 and 6 subtract 2 leaves the 4.

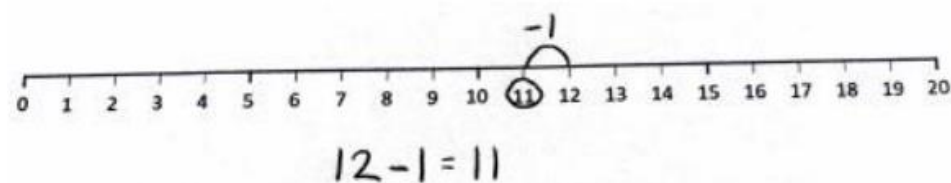
Stage 1

Represent number bonds within 10.

$$10 - 6 = 4$$

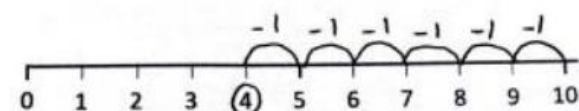


Find 1 less from any given number within 20.

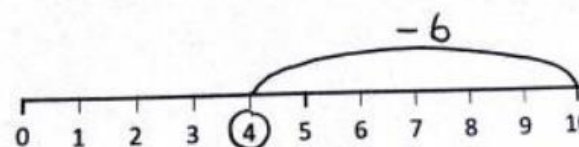


Stage 2

Use number bonds within 10.

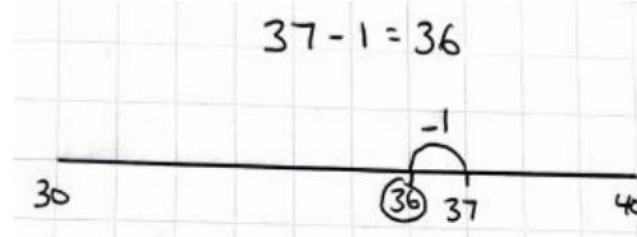


$$10 - 6 = 4$$



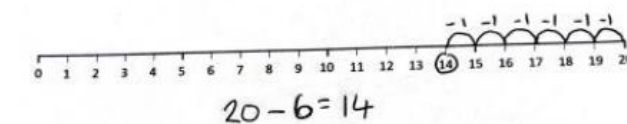
$$10 - 6 = 4$$

Find 1 less from any given number within 50.

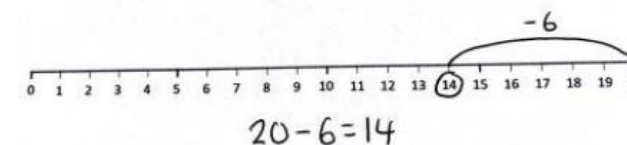


End of Year Expectation

Represent and use number bonds within 20.



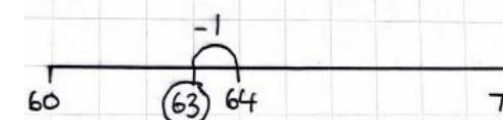
$$20 - 6 = 14$$



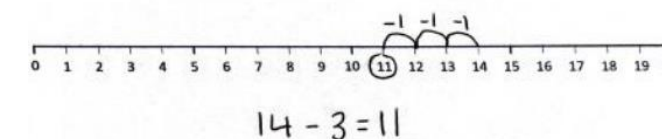
$$20 - 6 = 14$$

Find 1 less than any given number within 100.

$$64 - 1 = 63$$

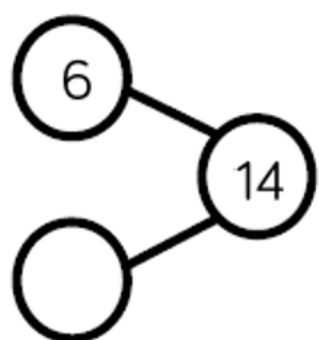


Subtract 1-digit and 2-digit numbers within 20.



$$14 - 3 = 11$$

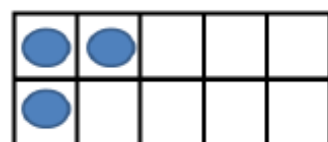
Year 1 Subtraction: Representations and Models



14

6

8

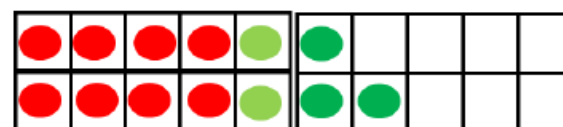


$$10 - \square = 3$$

$$20 = \square - \square$$

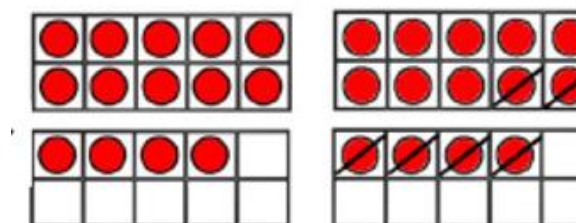
Using known facts

$$13 - 5 = 13 - 3 - 2$$



$$14 - 6 = 8$$

$$14 - 6 = 8$$



Number Facts – Year 2	
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Number and Place Value	
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- Know the sequence of counting in multiples of 3
- Count in steps of 10 from any number

Addition and Subtraction

- Know number bonds and related subtraction facts to 20
- Derive number bonds to 100 using multiples of 10, relating this to known number bonds to 10 (from Year 1)
- Add and subtract numbers to 100 using informal methods, manipulative resources and visual representations.

Multiplication and Division

- Know the 2, 5 and 10 x table and the related division facts
- Recognise odd and even numbers

Fractions

- $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 \text{ whole}$
- $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$
- $1 \text{ whole} - \frac{1}{4} = \frac{3}{4}$
- $\frac{2}{4} = \frac{1}{2}$
- Halve all even numbers to 20

Measure

- $100\text{p} = \text{£}1$
- $50\text{p} + 50\text{p} = \text{£}1$
- $100\text{ cm} = 1\text{ metre}$
- One hour = 60 minutes
- $\frac{1}{2}$ an hour = 30 minutes
- $\frac{1}{4}$ of an hour = 15 minutes
- $\frac{3}{4}$ of an hour = 45 minutes
- There are 24 hours in a day
- Recite the months of the year in correct order

Models and images to support conceptual understanding of year 2 number facts	100-square for skip counting in tens from any number
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to

Tens frames with counters and number lines to support subtracting ones from a multiple of 10

5

5

Three bags of five biscuits with three 5-value counters to support skip counting for $3 \times 5 = 15$

Number line for skip counting

28
20 8

28	
20	8

Partitioning 28 into 20 and 8

9

9

Half of 18 is 9

100-square for skip counting in tens from any number

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

ה

Diagram illustrating the addition of 45 and 30 using base ten blocks. The first box contains 4 tens rods and 5 ones units. The second box contains 3 tens rods. An arrow points to the third box, which contains 7 tens rods and 5 ones units, representing the sum 75.

	4	+	3	=	7
so	40	+	30	=	70
	45	+	30	=	75

Base 10 material and equations
to support adding a multiple of 10

Year 2: Addition

Linked to Hampshire Scheme of Learning Units 2.1, 2.2, 2.4, 2.5, 2.7, 2.8, 2.9 and 2.12.

National Curriculum Objectives:

Pupils should be taught to:

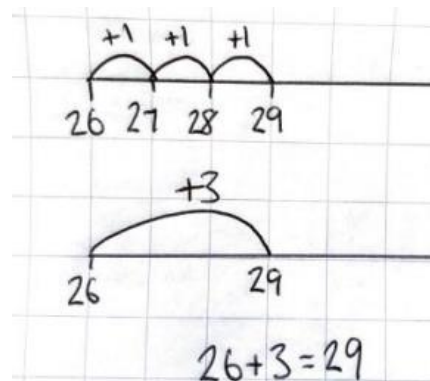
- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
 - a 2-digit number and ones
 - a 2-digit number and tens
 - two 2-digit numbers
 - adding three 1-digit numbers
- Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot

The Big Ideas:

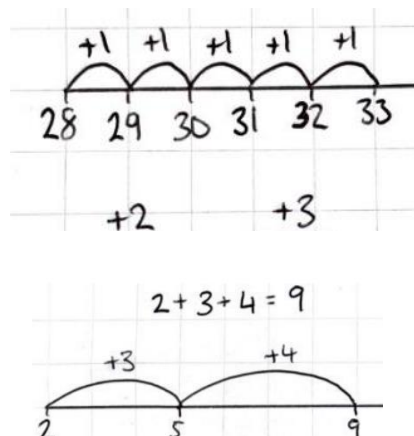
Understanding that addition of two or more numbers can be done in any order is important to support children's fluency. When adding two numbers it can be more efficient to put the larger number first. For example, given $3 + 8$ it is easier to calculate $8 + 3$. When adding three or more numbers it is helpful to look for pairs of numbers that are easy to add. For example, given $5 + 8 + 2$ it is easier to add $8 + 2$ first than to begin with $5 + 8$. Understanding the importance of the equals sign meaning 'equivalent to' (i.e. that $6 + 4 = 10$, $10 = 6 + 4$ and $5 + 5 = 6 + 4$ are all valid uses of the equals sign) is crucial for later work in algebra. Empty box problems can support the development of this key idea. Correct use of the equals sign should be reinforced at all times. Altering where the equals sign is placed develops fluency and flexibility.

Stage 1

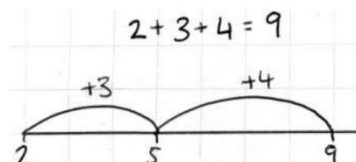
Add 2-digit numbers and ones to 50 without bridging.



Add 2-digit numbers and ones to 50 with bridging.

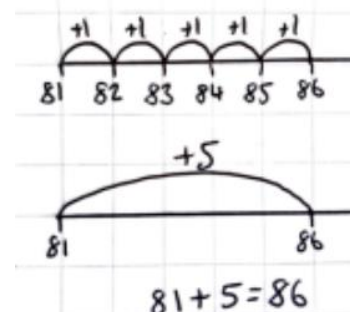


Adding three 1-digit numbers.

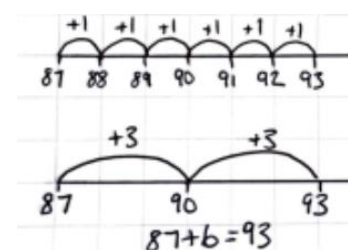


Stage 2

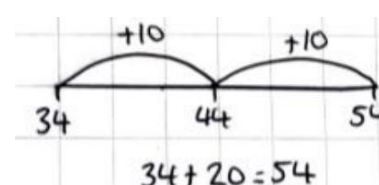
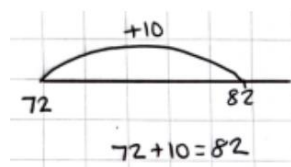
Add 2-digit numbers and ones to 100 without bridging.



Add 2-digit numbers and ones to 100 with bridging.

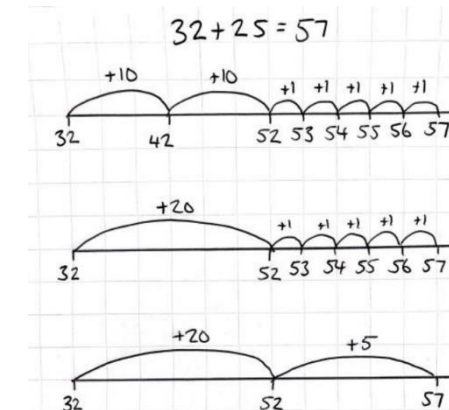


Add 2-digit and tens.

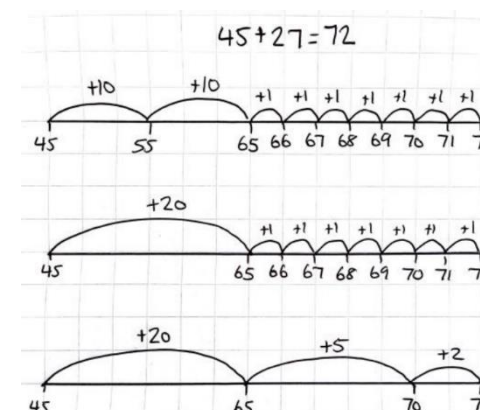


End of Year Expectation

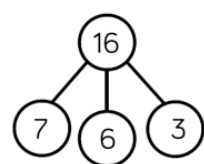
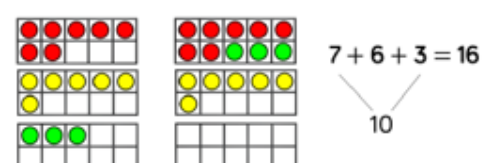
Adding two 2-digit numbers without bridging.



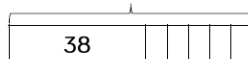
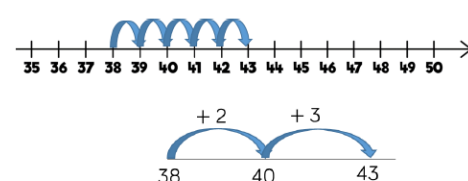
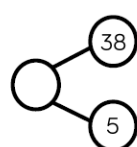
Adding two 2-digit numbers with bridging.



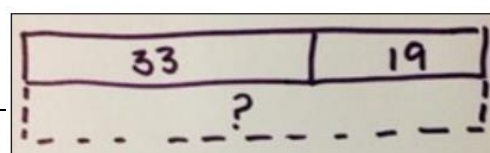
Year 2 Addition Representations and Models



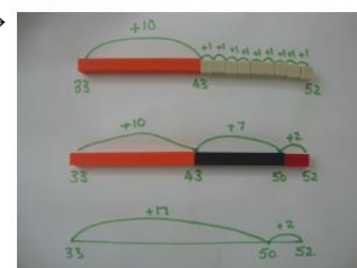
$$7 + 6 + 3 = 16$$



$$38 + 5 = 43$$



$$33 + 19 = ?$$

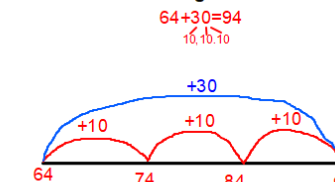


Round and adjust- adding near tens



33 + 19
33 + 10 + 7 + 2
33 + 17 + 2
Encourage use of number bonds
33 + 20 - 1

Number line - adding 10s



Progression

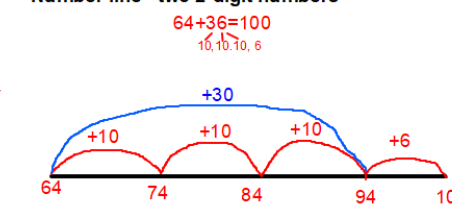
36 + 40 (adding only tens to any number)
36 + 43 (adding tens and ones with no bridging)
36 + 46 (adding tens and ones with bridging)

$$36 + 46$$

36 + 40 (36, 46, 56, 66, 76) add tens

76 + 6 or 76 + 4 + 2 (using number bond knowledge)

Number line - two 2-digit numbers



Year 2: Subtraction

Linked to Hampshire Scheme of Learning Units 2.1, 2.2, 2.4, 2.5, 2.7, 2.8, 2.9 and 2.12.

National Curriculum Objectives:

Pupils should be taught to:

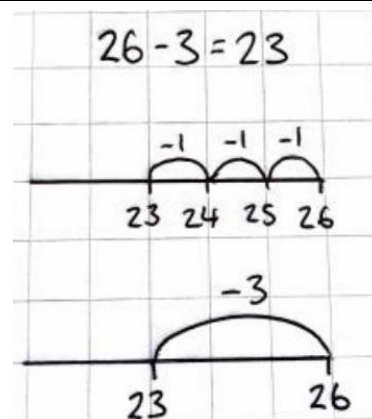
- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
 - a 2-digit number and ones
 - a 2-digit number and tens
 - two 2-digit numbers
 - adding three 1-digit numbers
- Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot

The Big Ideas:

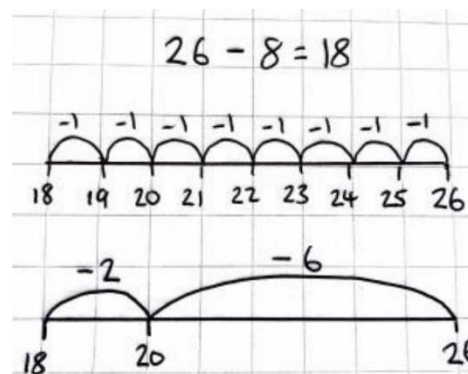
Relating numbers to 5 and 10 helps develop knowledge of the number bonds within 20. For example, given $8 + 7$, thinking of 7 as $2 + 5$, and adding the 2 and 8 to make 10, then the 5 to 15. This should then be applied when calculating with larger numbers. Subtraction bonds can be thought of in terms of addition: for example, in answering $15 - 8$, thinking what needs to be added to 8 to make 15. Counting on for subtraction is a useful strategy that can also be applied to larger numbers

Stage 1

Subtract 2-digit numbers and ones with numbers to 50 without bridging.

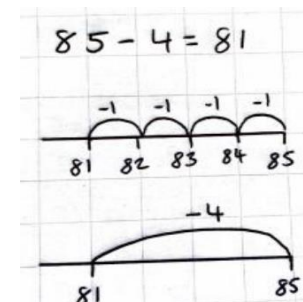


Subtract 2-digit numbers and ones to 50 with bridging.

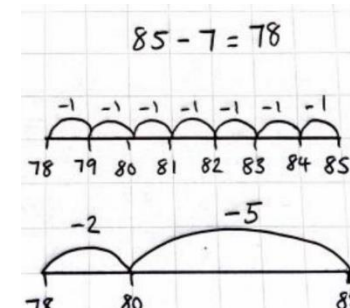


Stage 2

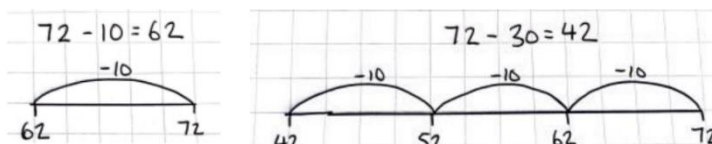
Subtract 2-digit numbers and ones to 100 without bridging.



Subtract 2-digit numbers and ones to 100 with bridging.

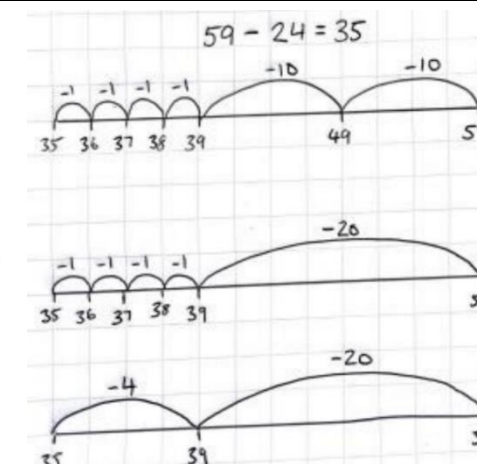


Subtract 2-digit numbers and tens.

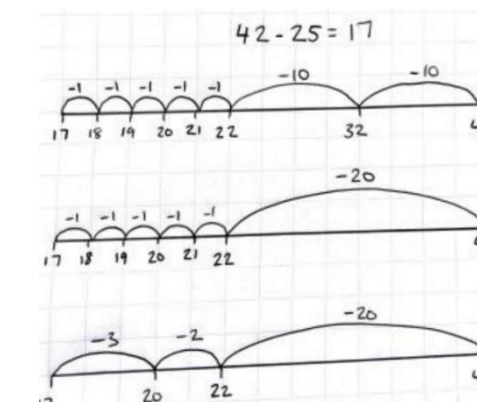


End of Year Expectation

Subtract two 2-digit numbers without bridging.



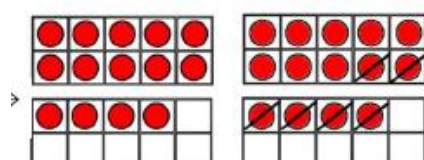
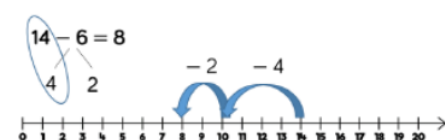
Subtract two 2-digit numbers with bridging



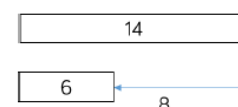
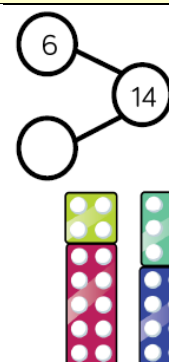
Year 2 Subtraction Representations and Models

Number line - subtracting 1 digit numbers

Count backwards from the larger number (whole) in efficient "chunks", using knowledge of number bonds of the smaller number (part).



$$14 - 6 = 8$$

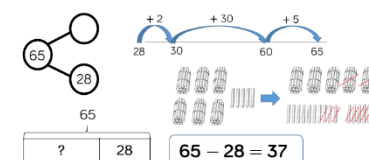
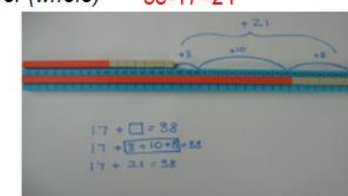


$$14 - 6 = 8$$



Number line - Finding the difference

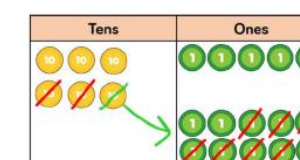
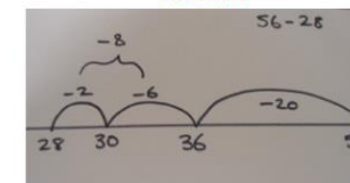
Start with the smaller number (part) and count on in manageable chunks until you reach the larger number (whole). $38 - 17 = 21$



Number line - subtracting 2 digit numbers

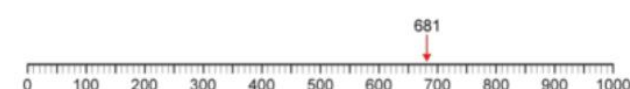
Put the whole at the end of the number line and count back in efficient "chunks"

$$56 - 28 = 28$$

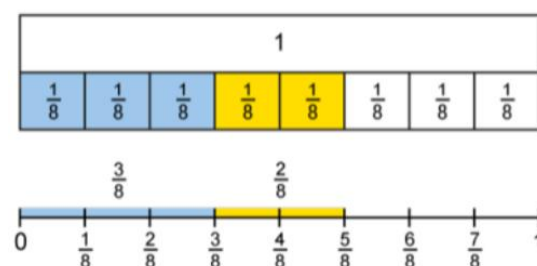


Number Facts – Year 3			
Number and Place Value	Addition and Subtraction	Fractions	Measure
<ul style="list-style-type: none"> Know the sequence of counting in 50s Know the sequence of counting in 100s 	<ul style="list-style-type: none"> Know or derive all the complements to 100 $x + y = 100$; $x = ?$ and $y = ?$ Know pairs of multiples of 100 that total 1000 $1 + 9 = 10$ (Year 1) $10 + 90 = 100$ (Year 2) $100 + 900 = 1000$ (Year 3) Add and subtract numbers with up to 3 digits (e.g. $253 + 75 = 328$) 	<ul style="list-style-type: none"> $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$ $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = 1$ whole $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$ whole $\frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = 1$ whole $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = 1$ whole $\frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = 1$ whole $\frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = 1$ whole Understand fraction facts related to whole numbers facts $1 + 5 = 6$ (Year 1) linked to $\frac{1}{6} + \frac{5}{6} = \frac{6}{6}$ (Year 3) 	<ul style="list-style-type: none"> 60 seconds = 1 minute How many days in each month/year/leap year Find complements to 60 50 p x 2 = £1 £50 x 2 = £1000 25 p x 4 = £1 £25 x 4 = £100 20p x 5 = £1 £20 x 5 = £100 1000 g = 1 kg 1000ml = 1l 1000 m = 1km 1000 ÷ 2 = 500 1000 ÷ 4 = 250 $\frac{1}{2}$ l/kg/km = 500 $\frac{1}{4}$ l/kg/km = 250 $\frac{3}{4}$ l/kg/km = 750
	Multiplication and Division <ul style="list-style-type: none"> Know the 3, 4 and 8 x table and the related division facts Understand that doubling means x2 Understand that halving means ÷ 2 Know that : 50 x 2 = 100 ; 25 x 4 = 100 ; 20 x 5 = 100 		

Models and images to support conceptual understanding of year 3 number facts



Number line to identify previous and next multiples of 100

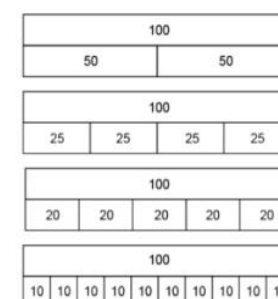


$$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$

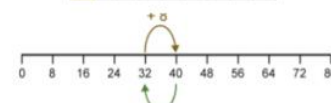
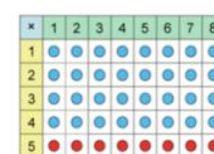
$$\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$$



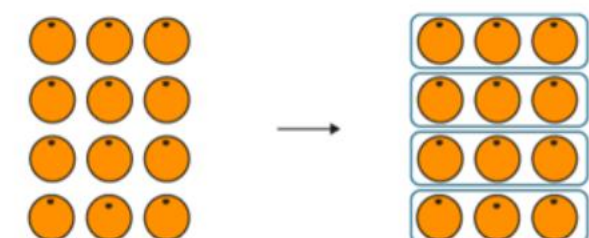
10-value place value counters in a 3-by-5 array to show $3 \times 50 = 30$ x 5 = 150



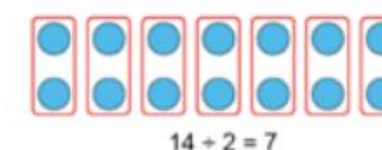
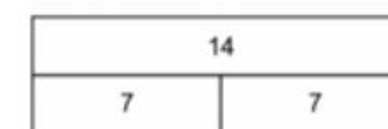
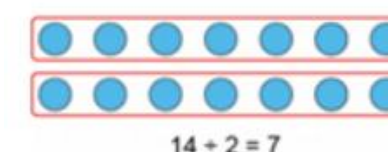
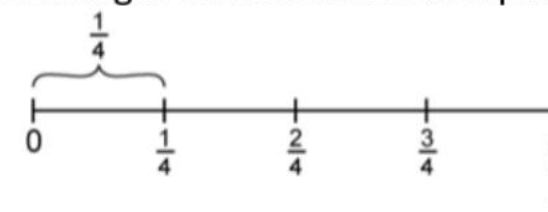
Bar models showing 100 partitioned into 2, 4, 5 and 10 equal parts.



Number line and array showing that adjacent multiples of 8 (32 and 40) have a difference of 8



12 oranges divided into four equal parts



$$7 \times 2 = 2 \times 7$$

Year 3: Addition

Linked to Hampshire Scheme of Learning Units 3.1, 3.2, 3.5, 3.7, 3.10 and 3.13.

National Curriculum Objectives:

Pupils should be taught to:

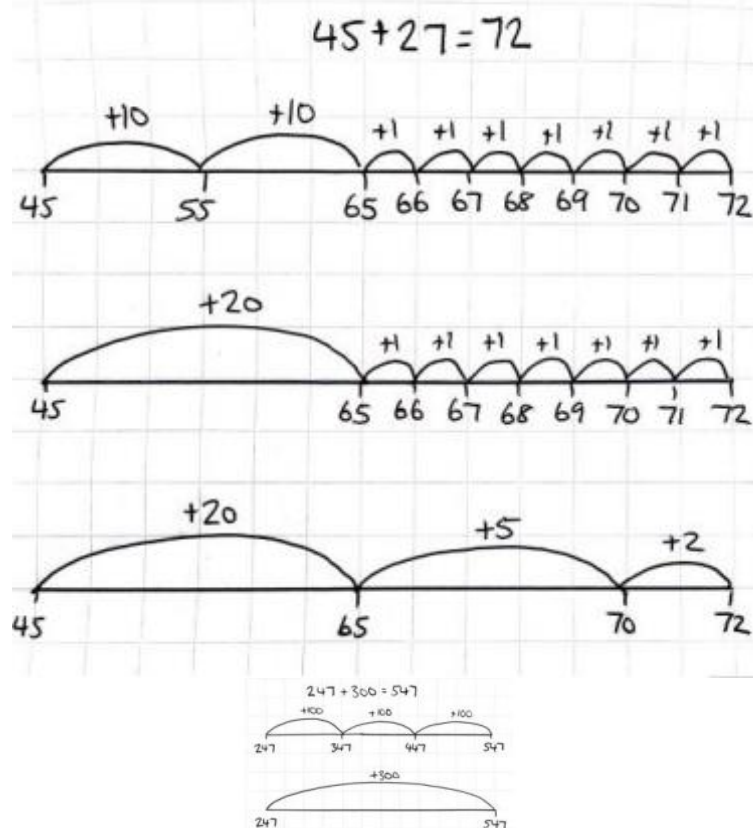
- Add and subtract numbers mentally, including:
 - a 3-digit number and ones
 - a 3-digit number and tens
 - a 3-digit number and hundreds
- Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

The Big Ideas:

Relating numbers to 5 and 10 helps develop knowledge of the number bonds within 20. For example, given $8 + 7$, thinking of 7 as 2 + 5, and adding the 2 and 8 to make 10, then the 5 to 15. This should then be applied when calculating with larger numbers. Subtraction bonds can be thought of in terms of addition: for example, in answering $15 - 8$, thinking what needs to be added to 8 to make 15. Counting on for subtraction is a useful strategy that can also be applied to larger numbers

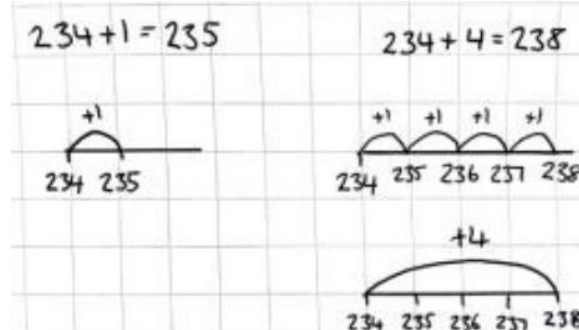
Stage 1

Adding two 2-digit numbers with bridging.

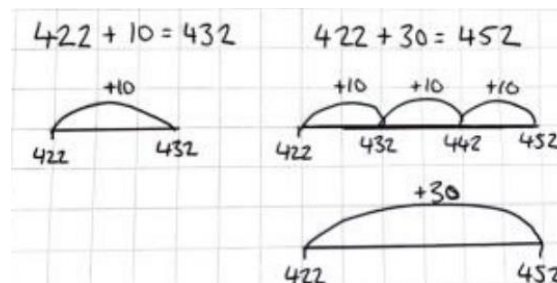


Stage 2

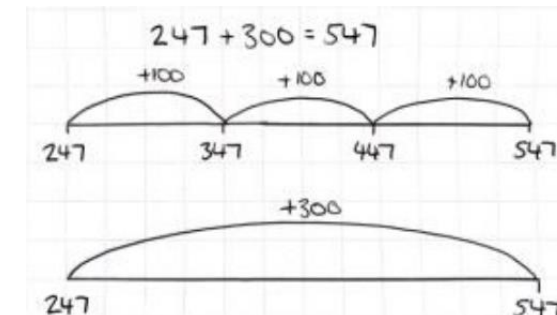
Adding a 3-digit number and ones.



Adding a 3-digit number and tens crossing the hundreds boundary.



Adding a 3-digit number and hundreds.



End of Year Expectation

Introduce column addition with numbers up to 3-digits.

Ensure number sentences chosen would not be more suited to a more efficient mental strategy.

Begin with expanded.

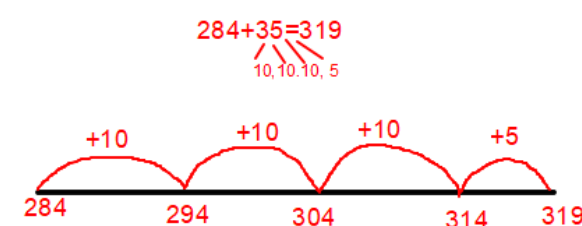
$$\begin{array}{r} 700 + 80 + 9 \\ 600 + 40 + 2 \\ \hline 1300 + 120 + 11 = 1431 \end{array}$$

Then progress to compact.

$$\begin{array}{r} 789 \\ + 642 \\ \hline 1431 \end{array}$$

Year 3 Addition Representations and Models

Number line for bridging 100s



Key Skills Progression

- Regrouping 10s
- Regrouping 100s/1000s
- Regrouping in more than 1 column

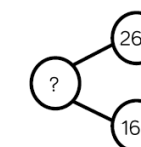
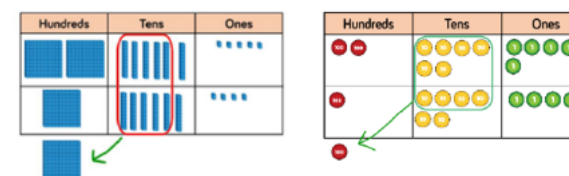
Expanded Column Addition

Begin adding in the ones column and the move up each column.

H	T	O
200	+	60
100	+	60
400	+	20
100	+	9

Representations alongside expanded column

$$265 + 164 = 429$$



$$\begin{array}{r} 265 \\ + 164 \\ \hline \end{array}$$

$$\begin{array}{r} 265 \\ + 164 \\ \hline \end{array}$$

$$265 + 164 = 429$$

Year 3: Subtraction

Linked to Hampshire Scheme of Learning Units 3.1, 3.2, 3.5, 3.7, 3.10 and 3.13

National Curriculum Objectives:

Pupils should be taught to:

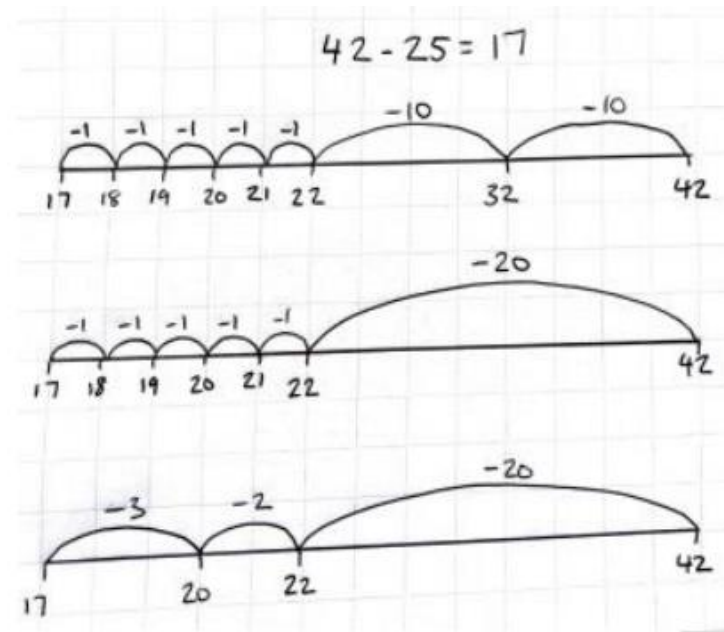
- Add and subtract numbers mentally, including:
 - a 3-digit number and ones to a 3-digit number and tens
 - a 3-digit number and hundreds
- Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

The Big Ideas:

Relating numbers to 5 and 10 helps develop knowledge of the number bonds within 20. For example, given $8 + 7$, thinking of 7 as 2 + 5, and adding the 2 and 8 to make 10, then the 5 to 15. This should then be applied when calculating with larger numbers. Subtraction bonds can be thought of in terms of addition: for example, in answering $15 - 8$, thinking what needs to be added to 8 to make 15. Counting on for subtraction is a useful strategy that can also be applied to larger numbers

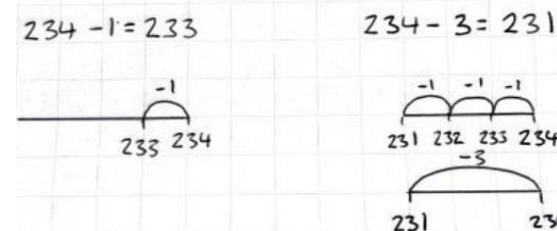
Stage 1

Subtract two 2-digit numbers with bridging.

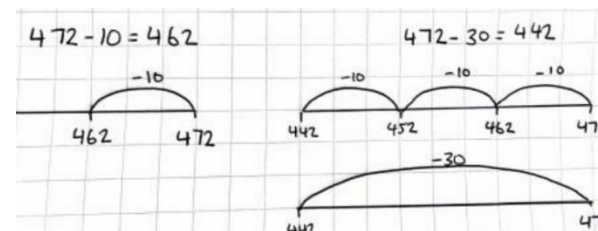


Stage 2

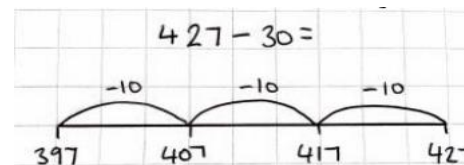
Subtract a 3-digit number and ones.



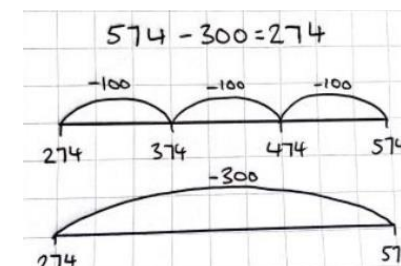
Subtract a 3-digit number and tens.



Subtract a 3-digit number and tens crossing the hundreds boundary.



Subtract a 3-digit number and hundreds



End of Year Expectation

Introduce column subtraction with numbers up to 3-digits.

Ensure number sentences chosen would not be more suited to a more efficient mental strategy.

Begin with expanded (without carrying).

$$\begin{array}{r} 900 - 60 - 7 \\ 400 - 50 - 2 \\ 500 - 10 - 5 = 515 \end{array}$$

Progress to compact (without carrying then with carrying).

$$\begin{array}{r} 892 \\ - 457 \\ \hline 435 \end{array}$$

Year 3 Subtraction Representations and Models

Expanded Column Subtraction

Begin with no exchanging. Start subtracting in the ones column and the move up each column.

$$265 - 134 = 131$$

H	T	O
200	60	5
100	30	4
100	30	1

Expanded Column Subtraction

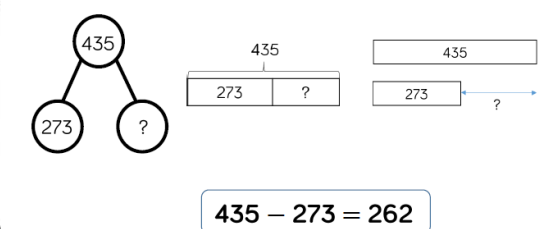
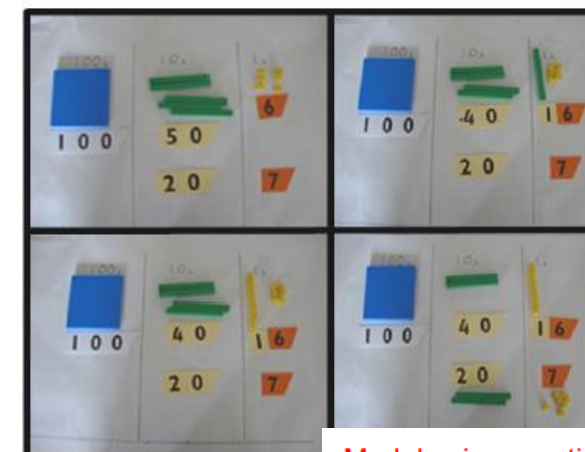
Start subtracting in the ones column and the move up each column.

$$435 - 273 = 162$$

H	T	O
400	30	5
200	70	3
100	60	2

Representations alongside expanded column

$$435 - 273 = 162$$



Key skills progression—exchanging or regrouping

No regrouping/exchanging required

Regrouping tens (exchanging from 1s for 10s only)

Regrouping from hundreds only (exchanging from 10s)

Regrouping in more than 1 column (e.g. exchanging both 1s and 10s for 100s)

Model using practical apparatus alongside written methods so children UNDERSTAND what is happening.

Number Facts- Year 4			
Number and Place Value	Addition and Subtraction	Fractions	Measure
<ul style="list-style-type: none"> Know the sequence of counting in multiples of 25 	<ul style="list-style-type: none"> Know or derive all the complements to 100 $x + y = 10\ 000$; $x = ?$ and $y = ?$ Know pairs of multiples of 100 that total 1000 $1 + 9 = 10$ (Year 1) $10 + 90 = 100$ (Year 2) $100 + 900 = 1000$ (Year 3) $1000 + 9000 = 10\ 000$ (Year 4) Mentally add and subtract numbers with up to 2 digits reliably 	<ul style="list-style-type: none"> $100 \div 10 = 10$ $1000 \div 10 = 100$ $10 \div 10 = 1$ $1 \div 10 = \frac{1}{10}$ $1 \div 10 = \frac{1}{10} = 0.1$ $2 \div 10 = \frac{2}{10} = 0.2$ $3 \div 10 = \frac{3}{10} = 0.3$ $4 \div 10 = \frac{4}{10} = 0.4$ $5 \div 10 = \frac{5}{10} = 0.5$ $6 \div 10 = \frac{6}{10} = 0.6$ $7 \div 10 = \frac{7}{10} = 0.7$ $8 \div 10 = \frac{8}{10} = 0.8$ $9 \div 10 = \frac{9}{10} = 0.9$ $10 \div 10 = \frac{10}{10} = 1.0$ $\frac{1}{4} = 0.25$ $\frac{1}{2} = 0.5$ $\frac{3}{4} = 0.75$ 	<ul style="list-style-type: none"> $£ 5 \times 2 = £10$ $£50 \times 2 = £100$ $£500 \times 2 = £1000$ $£2.50 \times 4 = £10$ $£25 \times 4 = £100$ $£250 \times 4 = £1000$ $£2 \times 5 = £10$ $£20 \times 5 = £100$ $£200 \times 5 = £1000$ $10\text{cm} = \frac{1}{10} \text{ m}$ $1\text{cm} = \frac{1}{100} \text{ m}$ $100\text{g} = \frac{1}{10} \text{ kg}$ $1.1\text{kg} = 1\text{kg} \text{ and } 100\text{g} = 1\text{kg} + \frac{1}{10} \text{ kg}$ $48 \text{ hours} = 2 \text{ days}$ $120 \text{ minutes} = 2 \text{ hours}$ $90 \text{ minutes} = 1 \frac{1}{2} \text{ hours}$
	Multiplication and Division <ul style="list-style-type: none"> Know the 6, 7, 9, 11 and 12 x tables and the related division facts Know that: $500 \times 2 = 1000$ $1000 \div 2 = 500$ $250 \times 4 = 1000$ $1000 \div 4 = 250$ $200 \times 5 = 1000$ $1000 \div 5 = 200$ 		

Models and images to support conceptual understanding of year 4 number facts

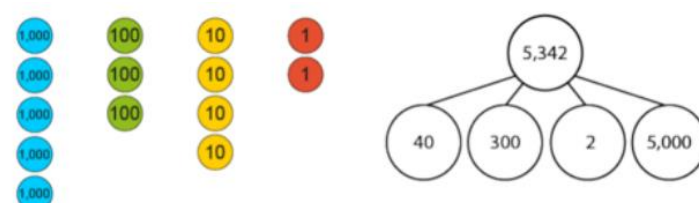


eighteen 100-value place-value counters in two tens frames to show 1800

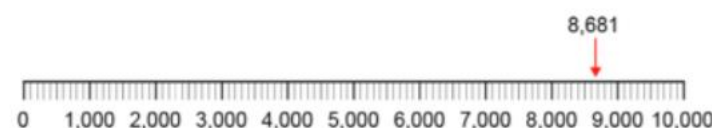
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

$80 \times 10 = 800$ $80 \div 10 = 8$

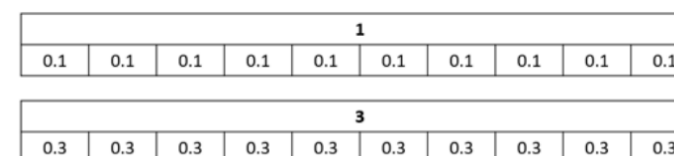
Gattegno chart to multiply and divide by 10



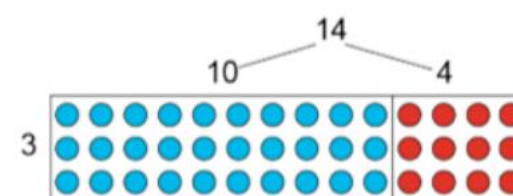
Representations of the place value composition of 5,342



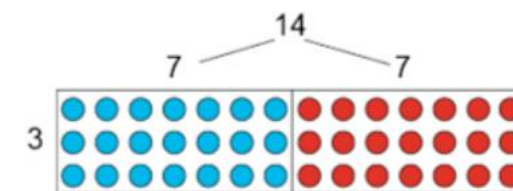
number-line to identify the previous and next multiple of 1,000



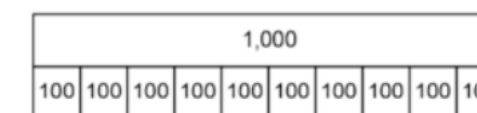
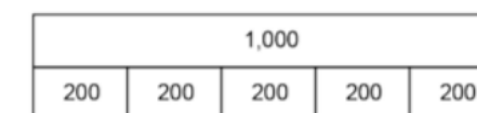
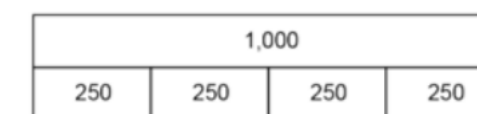
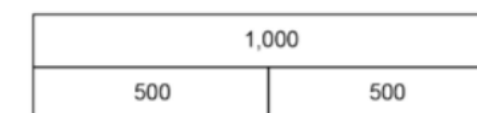
bar models showing $1 \div 10 = 0.1$ and $3 \div 10 = 0.3$



array to show that $14 \times 3 = 10 \times 3 + 4 \times 3$



array to show that $14 \times 3 = 2 \times 7 \times 3$



bar models showing 1,000 partitioned into 2, 4, 5, and 10 equal parts and

$1000 \div 2 = 500$ and $\frac{1}{2}$ of 1000 = 500
 $1000 \div 4 = 250$ and $\frac{1}{4}$ of 1000 = 250
 $1000 \div 5 = 200$ and $\frac{1}{5}$ of 1000 = 200
 $1000 \div 10 = 100$ and $\frac{1}{10}$ of 1000 = 100

Year 4: Addition

Linked to Hampshire Scheme of Learning Units 4.1, 4.2, 4.7, 4.10 and 4.13

National Curriculum Objectives:

Pupils should be taught to:

- Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- Solve addition and subtraction two-step problems in context, deciding which operations and methods to use and why
- Count backwards through zero to include negative numbers (subtraction only)

The Big Ideas:

It helps to round numbers before carrying out a calculation to get a sense of the size of the answer. For example, $4786 - 2135$ is close to $5000 - 2000$, so the answer will be around 3000. Looking at the numbers in a calculation and their relationship to each other can help make calculating easier. For example, $3012 - 2996$. Noticing that the numbers are close to each other might mean this is more easily calculated by thinking about subtraction as difference.

Stage 1

Column addition with numbers up to 3-digits.

Ensure number sentences chosen would not be more suited to an efficient strategy.

$$\begin{array}{r} 789 \\ + 642 \\ \hline 1431 \\ \hline 11 \end{array}$$

Stage 2

Column addition with numbers up to 4-digits.

Ensure number sentences chosen would not be more suited to an efficient strategy.

$$\begin{array}{r} 3982 \\ + 1766 \\ \hline 5748 \\ \hline 11 \end{array}$$

End of Year Expectation

Multistep problem in context involving addition.

(Provide a mixture of opportunities to apply mental and formal strategies taught)

Some children vote for their favourite ice-cream flavour.

Ice-cream flavour	Number of children
vanilla	87
chocolate	154
strawberry	?
mint	38
Total	402

How many children vote for **strawberry**?

$$\begin{array}{r} 154 \\ + 87 \\ \hline 241 \\ \hline 11 \end{array} \quad \begin{array}{r} 241 \\ + 38 \\ \hline 279 \\ \hline 11 \end{array}$$

Year 4 Addition Representations and Models

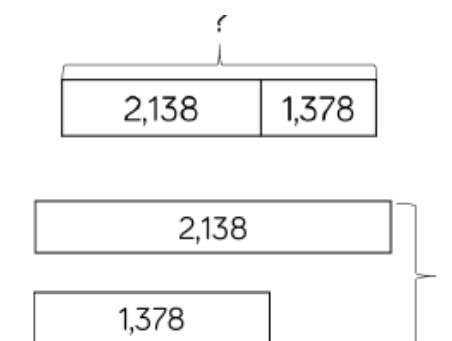
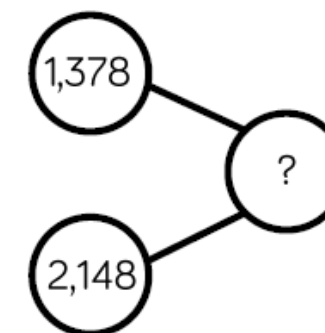
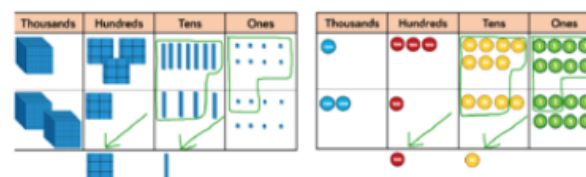
Compact Column Addition

Starting adding in the ones column and move up each column in order.

$$\begin{array}{r} 1378 \\ + 2148 \\ \hline 3526 \\ \hline 11 \end{array}$$

Representations alongside expanded column

$$1,378 + 2,148 = 3,526$$



Year 4: Subtraction

Linked to Hampshire Scheme of Learning Units 4.1, 4.2, 4.7, 4.10 and 4.13

National Curriculum Objectives:

Pupils should be taught to:

- Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- Solve addition and subtraction two-step problems in context, deciding which operations and methods to use and why
- Count backwards through zero to include negative numbers (subtraction only)

The Big Ideas:

It helps to round numbers before carrying out a calculation to get a sense of the size of the answer. For example, $4786 - 2135$ is close to $5000 - 2000$, so the answer will be around 3000. Looking at the numbers in a calculation and their relationship to each other can help make calculating easier. For example, $3012 - 2996$. Noticing that the numbers are close to each other might mean this is more easily calculated by thinking about subtraction as difference.

Stage 1

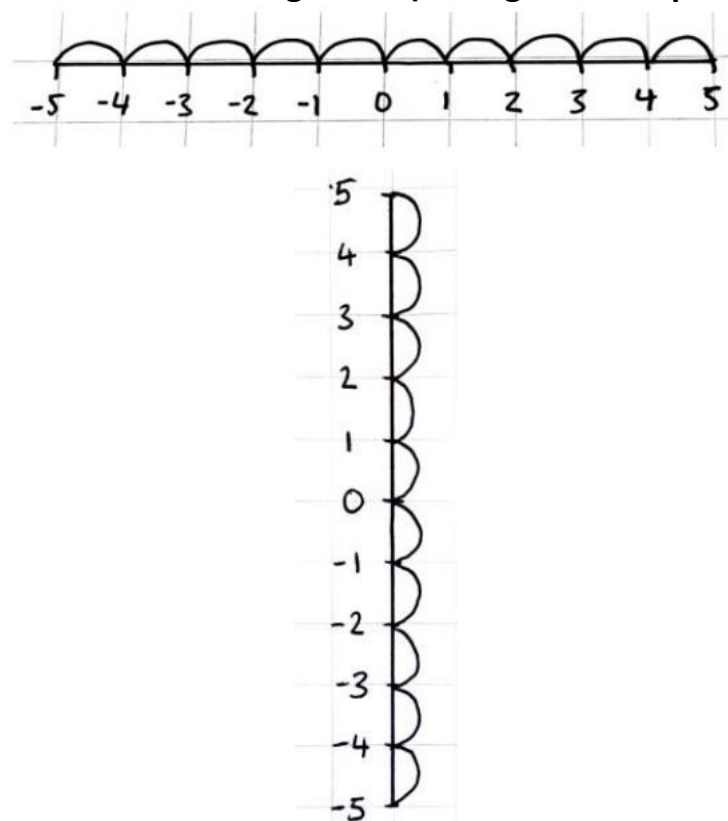
Column subtraction up to 3-digits.

Ensure number sentences chosen would not be more suited to a more efficient mental strategy.

$$\begin{array}{r} 8 \quad 12 \quad 12 \\ 457 \\ - 457 \\ \hline 475 \end{array}$$

Stage 2

Count backwards through zero (linking with temperature).



End of Year Expectation

Column subtraction with numbers up to 4-digits.

Ensure number sentences chosen would not be more suited to a more efficient mental strategy.

$$\begin{array}{r} 18 \quad 14 \\ 1949 \\ - 1949 \\ \hline 925 \end{array}$$

Year 4 Subtraction Representations and Models

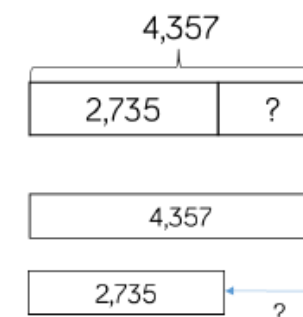
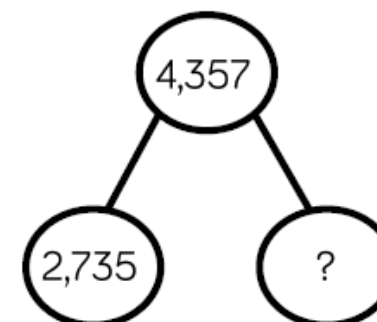
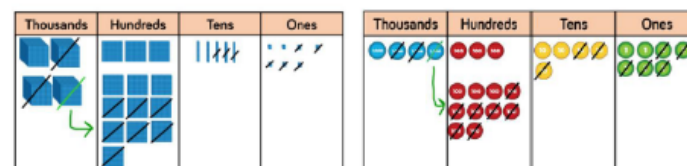
Compact Column Subtraction

Starting subtracting in the ones column and move up each column in order.

$$\begin{array}{r} 3 \quad 1 \\ 4357 \\ - 2735 \\ \hline 1622 \end{array}$$

Representations alongside expanded column

$$4,357 - 2,735 = 1,622$$



Number Facts- Year 5			
Geometry	Addition and Subtraction Multiplication and Division	Fractions	Measure
<ul style="list-style-type: none"> $360 \div 4 = 90$ $\frac{1}{4}$ of 360 = 90 $360 \div 2 = 180$ $\frac{1}{2}$ of 360 = 180 $\frac{3}{4}$ of 360 = 270 Complements such as $70 + 110 = 180$ $95 + 85 = 180$ Multiples: 90, 180, 270, 360, 450, 540 	<ul style="list-style-type: none"> Derive new facts from known facts. For example: $12 \times 5 = 60$ $60 \div 5 = 12$ $5.2 \times 5 = 6.0$ $6 \div 5 = 1.2$ $5 \times 7 = 35$ $5 \times 0.7 = 3.5$ $5 \times 0.07 = 0.35$ Square numbers: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144 Prime numbers: 2, 3, 5, 7, 11, 17, 19 Associated Facts: $10,000 = 9500 + 500$ $10,000 = 5000 + 5000$ $10,000 = 2500 + 2500 + 2500 + 2500$ $10,000 \div 2 = 5000$ $10,000 \div 4 = 2500$ $10,000 \div 5 = 2000$ $10,000 \div 10 = 1000$ $10,000 \div 100 = 100$ 	<ul style="list-style-type: none"> $1 \div 100 = \frac{1}{100} = 0.01$ $2 \div 100 = \frac{2}{100} = 0.02$ $3 \div 100 = \frac{3}{100} = 0.03$ $4 \div 100 = \frac{4}{100} = 0.04$ $5 \div 100 = \frac{5}{100} = 0.05$ $6 \div 100 = \frac{6}{100} = 0.06$ $7 \div 100 = \frac{7}{100} = 0.07$ $8 \div 100 = \frac{8}{100} = 0.08$ $9 \div 100 = \frac{9}{100} = 0.09$ $10 \div 100 = \frac{10}{100} = 0.1$ $10\% = 0.1 = \frac{1}{10} = \frac{10}{100} = \frac{100}{1000}$ $50\% = 0.5 = \frac{5}{10} = \frac{50}{100}$ $25\% = 0.25 = \frac{1}{4} = \frac{25}{100}$ $75\% = 0.75 = \frac{3}{4} = \frac{75}{100}$ $20\% = 0.2 = \frac{1}{5} = \frac{2}{10} = \frac{20}{100}$ $40\% = 0.4 = \frac{2}{5} = \frac{4}{10} = \frac{40}{100}$ 	<ul style="list-style-type: none"> $1\text{mm} = \frac{1}{10} \text{ cm}$ $1\text{mm} = \frac{1}{1000} \text{ m}$ $1\text{kg} \approx 2.2\text{lbs}$ $1\text{l} \approx 1.76 \text{ pints}$ $1\text{m} \approx 39.4 \text{ inches}$ $1\text{cm} \approx 2.54 \text{ inches}$

Models and images to support conceptual understanding of year 5 number facts

Square numbers have an odd number of factors

Prime numbers have exactly two factors

A hundred grid divided into four equal parts.

$\frac{1}{4} = 25\%$

Ratio tables for conversion

1m	100cm	1,000ml	1 litre	100p	£1
$\frac{3}{4}$ m	75cm	3,700ml	3.7 litres	52p	£0.52

Bar models showing 1 partitioned into 2, 4, 5 and 10 equal parts

$1 \div 2 = 0.5$ and $\frac{1}{2}$ of 1 = 0.5
 $1 \div 4 = 0.25$ and $\frac{1}{4}$ of 1 = 0.25
 $1 \div 5 = 0.2$ and $\frac{1}{5}$ of 1 = 0.2
 $1 \div 10 = 0.1$ and $\frac{1}{10}$ of 1 = 0.1

Multiplicative relationships between powers of ten

Using a number track to generate multiples of primes to identify primes: 2, 3, 5, 7, 11, 13, 17, 19

Year 5: Addition

Linked to Hampshire Scheme of Learning Units 5.1, 5.7, 5.10, 5.14, 5.15 and 5.18

National Curriculum Objectives:

- Add and subtract whole numbers with more than four digits, including using formal written methods (columnar addition and subtraction)
- Solve problems involving numbers up to three decimal places.
- Interpret negative numbers in context, count forwards and backwards with positive and negative numbers through zero

The Big Ideas

Before starting any calculation is it helpful to think about whether or not you are confident that you can do it mentally. For example, $3689 + 4998$ may be done mentally, but $3689 + 4756$ may require paper and pencil. Carrying out an equivalent calculation might be easier than carrying out the given calculation. For example, $3682 - 2996$ is equivalent to $3686 - 3000$ (constant difference).

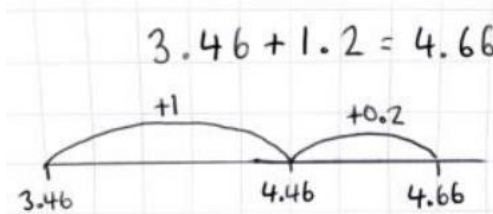
Stage 1

Column addition with numbers up to 4-digits.

Ensure number sentences chosen would not be more suited to an efficient strategy.

Handwritten column addition of 3982 + 1766 = 5748. The calculation is shown on a grid with carry marks (1, 1) under the tens and hundreds columns.

Adding involving numbers up to 1 and 2 decimal places.



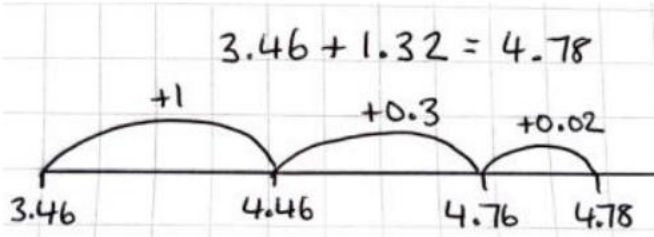
Stage 2

Column addition with numbers with more than 4-digits.

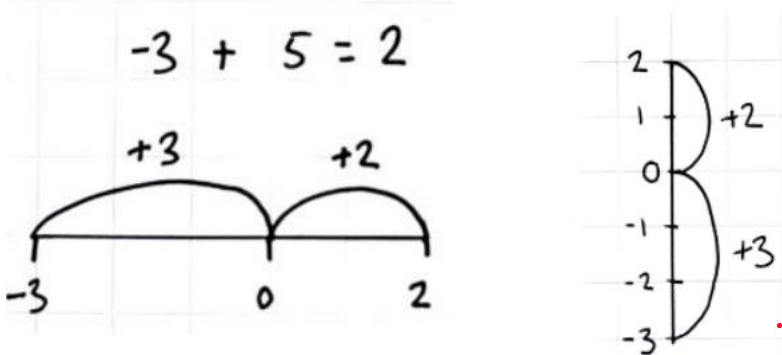
Ensure number sentences chosen would not be more suited to an efficient strategy.

Handwritten column addition of 23972 + 45639 = 69611. The calculation is shown on a grid with carry marks (1, 1, 1) under the tens, hundreds, and thousands columns.

Adding involving numbers up to 2 decimal places.



Adding with negative numbers (linking with temperature).



End of Year Expectation

Multistep problem in context, deciding which methods to use and why.

Provide a mixture of opportunities to apply mental and formal strategies taught

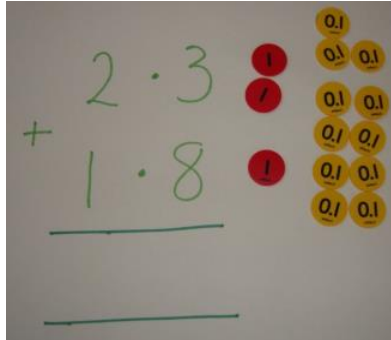
This table shows the number of people living in various towns in England.

Town	Population
Bedford	82,448
Carlton	48,493
Dover	34,087
Formby	24,478
Telford	166,640

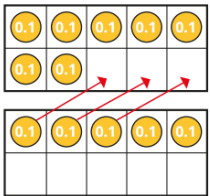
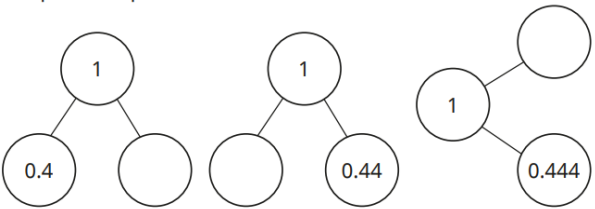
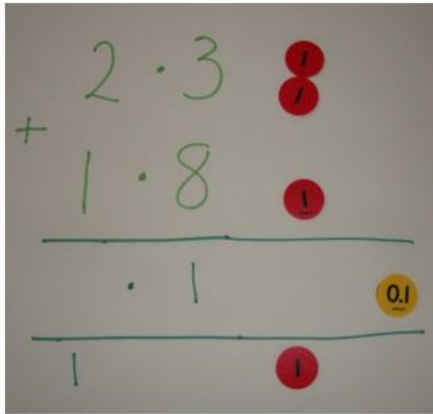
What is the total of the numbers of people living in Formby and in Telford?

Handwritten column addition of 24,478 + 66,640 = 91,118. The calculation is shown on a grid with carry marks (1, 1, 1) under the tens, hundreds, and thousands columns.

Year 5 Addition Representations and Models



Add up tenths first then whole numbers.



$0.7 + 0.3 = 1$
 $1 + 0.2 = 1.2$
 $0.7 + 0.5 = 1.2$

TTh	Th	H	T	O
●●●●	●●●●●●●●	●●●●●●●●	●●●●●●●●	

- ▶ 48,650 - 3,000
- ▶ 48,650 - 3,210
- ▶ 43,650 - 200
- ▶ 48,650 - 7,100
- ▶ 43,650 - 10
- ▶ 48,650 - 5,030

Year 5 Subtraction

Linked to Hampshire Scheme of Learning Units 5.1, 5.7, 5.10, 5.14, 5.15 and 5.18

National Curriculum Objectives:

Pupils should be taught to:

- Add and subtract whole numbers with more than four digits, including using formal written methods (columnar addition and subtraction)
- Solve problems involving numbers up to three decimal places
- Interpret negative numbers in context, count forwards and backwards with positive and negative numbers through zero.

The Big Ideas

Before starting any calculation it is helpful to think about whether or not you are confident that you can do it mentally. For example, $3689 + 4998$ may be done mentally, but $3689 + 4756$ may require paper and pencil. Carrying out an equivalent calculation might be easier than carrying out the given calculation. For example, $3682 - 2996$ is equivalent to $3686 - 3000$ (constant difference). -3000 (constant difference).

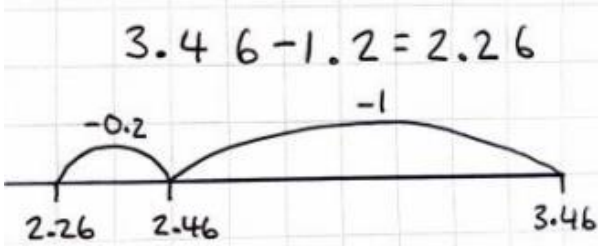
Stage 1

Column subtraction with numbers up to 4-digits.

Ensure number sentences chosen would not be more suited to a more efficient mental strategy.

$$\begin{array}{r} 1874 \\ - 949 \\ \hline 925 \end{array}$$

Subtracting involving numbers up to 1 and 2 decimal places.



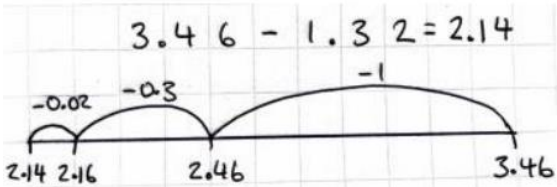
Stage 2

Column subtraction with numbers up to 5-digits.

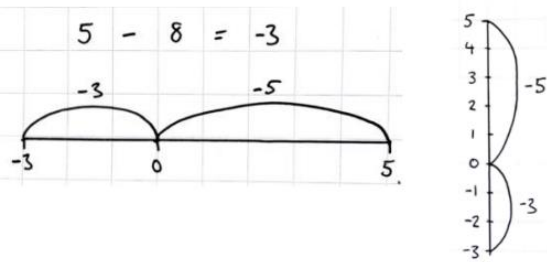
Ensure number sentences chosen would not be more suited to a more efficient mental strategy.

$$\begin{array}{r} 7482 \\ - 3329 \\ \hline 4163 \end{array}$$

Subtracting involving numbers up to 2 decimal places.



Subtract with negative numbers (linking with temperature).



End of Year Expectation

Multistep problem in context, deciding which methods to use and why.

Provide a mixture of opportunity to apply mental and formal strategies taught

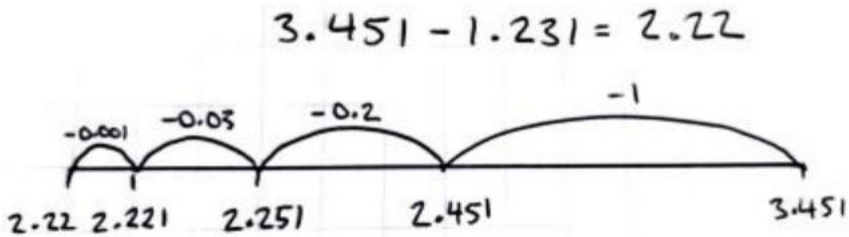
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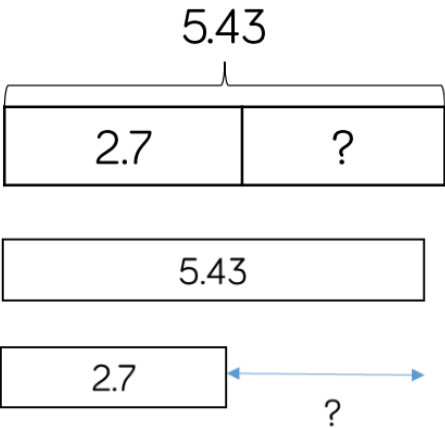
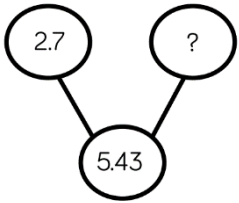
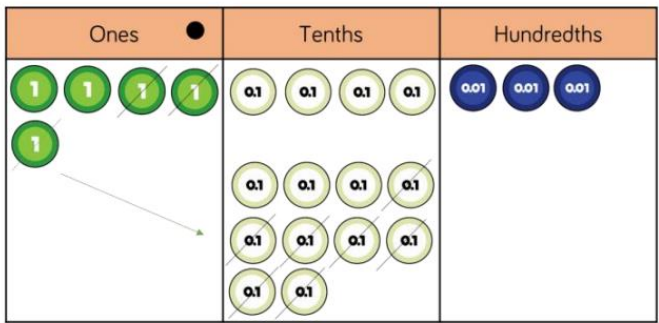
What is the difference between the numbers of people living in Bedford and in Dover?

$$\begin{array}{r} 812348 \\ - 34087 \\ \hline 48361 \end{array}$$

Subtracting involving numbers up to 3 decimal places.



Year 5 Subtraction Representations and Models

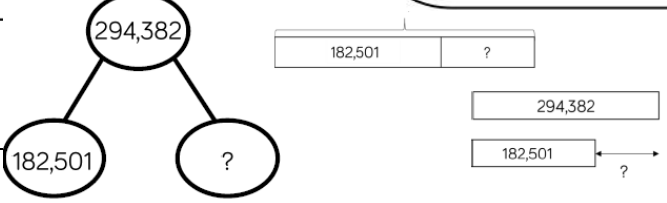


Compact Column Subtraction

Starting subtracting in the ones column and move up each column in order.

$$294,382 - 182,501 = 111,881$$

Representations alongside column subtraction

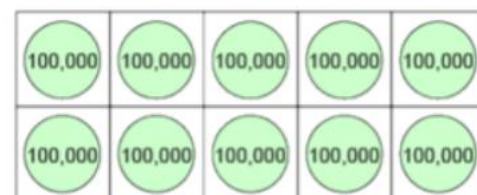


Number Facts- Year 6

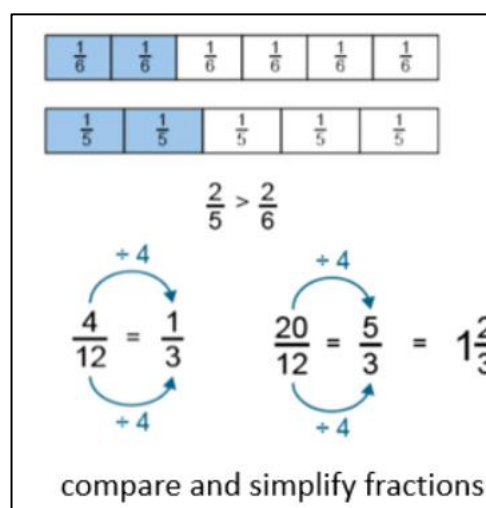
Ratio and Proportion	Geometry	Fractions	Measure
<ul style="list-style-type: none"> Derive new % facts from known facts: For example: 1% doubled will give 2% of a quantity 10% halved will give 5% of a quantity 100% is the whole amount, so twice as much is the same as 200% Fluency with multiplication and division facts up to 12 x 12 and derive others beyond known facts. For example: 24 : 48 simplifies to 1:2 with a common factor of 24 (24 x 1 and 24 x 2) 	<ul style="list-style-type: none"> Diameter = 2 x radius Radius = $\frac{1}{2}$ x diameter 	<ul style="list-style-type: none"> 12.5% = $0.125 = \frac{1}{8}$ 25% = $0.25 = \frac{2}{8} = \frac{1}{4}$ 37.5% = $0.375 = \frac{3}{8}$ 50% = $0.5 = \frac{4}{8} = \frac{1}{2}$ 62.5% = $0.625 = \frac{5}{8}$ 75% = $0.75 = \frac{6}{8} = \frac{3}{4}$ 82.5% = $0.825 = \frac{7}{8}$ 100% = $1.0 = \frac{8}{8}$ 112.5% = $1.125 = \frac{9}{8}$ 125% = $1.25 = \frac{10}{8}$ 33.3% = $0.333... = \frac{1}{3}$ 66.6% = $0.666... = \frac{2}{3}$ 100% = $1.0 = \frac{3}{3}$ 133.3% = $1.333... = \frac{4}{3}$ 266.6% = $2.666... = \frac{8}{3}$ $0.\dot{3} = 0.3333333...$ a recurring decimal continually repeats and does not terminate 	<ul style="list-style-type: none"> 1 km $\approx \frac{5}{8}$ mile 1 mile $\approx \frac{8}{5}$ km (or 1.6 km) Area of a triangle = $\approx \frac{1}{2}$ base x height Area of a rectangle = length x width Area of a parallelogram = length x perpendicular height Volume of a cuboid = length x width x height

Models and images to support conceptual understanding of year 6 number facts

0.01	one hundredth
0.1	one tenth
1	one
10	ten
100	one hundred
1,000	one thousand
10,000	ten thousand
100,000	one hundred thousand
1,000,000	one million
10,000,000	ten million



One million represented as ten 100,000-value place-value counters in a tens frame



bead strings to show 'for every 1 red bead, there are 3 blue beads'
 $r : b = 1 : 3$

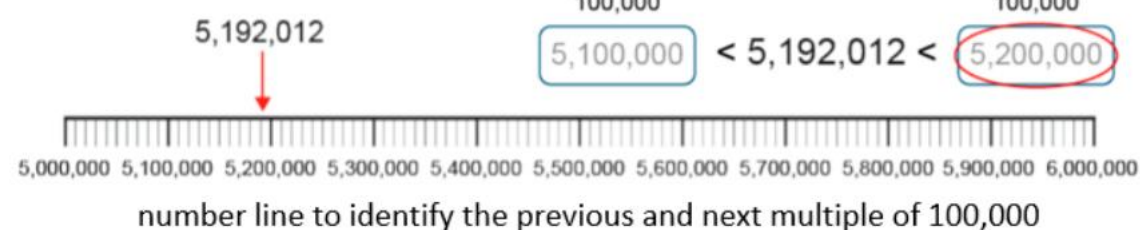
table to show total quantities in proportion

number of red beads	1	2	3	4
number of blue beads	3	6	9	12
total number of beads	4	8	12	16

1,000,000			
250,000	250,000	250,000	250,000

1,000			
250	250	250	250

1			
0.25	0.25	0.25	0.25



Year 6 Addition

Linked to Hampshire Scheme of Learning Units 6.1, 6.7, 6.10 and 6.15

National Curriculum Objectives:

Pupils should be taught to:

- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- Use negative numbers in context, and calculate intervals across zero

The Big Ideas

Deciding which calculation method to use is supported by being able to take apart and combine numbers in many ways. For example, calculating $8 \cdot 78 + 5 \cdot 26$ might involve calculating $8 \cdot 75 + 5 \cdot 25$ and then adjusting the answer. The associative rule helps when adding three or more numbers: $367 + 275 + 525$ is probably best thought of as $367 + (275 + 525)$ rather than $(367 + 275) + 525$.

Stage 1

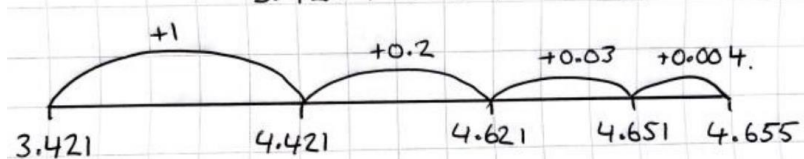
Column addition with numbers with more than 4-digits.

Ensure number sentences chosen would not be more suited to an efficient strategy.

$$\begin{array}{r} 23972 \\ + 45639 \\ \hline 69611 \end{array}$$

Adding involving numbers up to 3 decimal places.

$$3.421 + 1.234 = 4.655$$



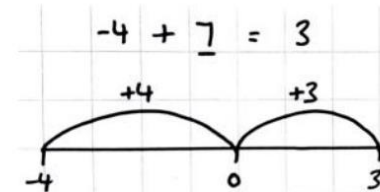
Stage 2

Adding negative numbers in context.

City	Temperature	
	At midnight	At midday
Paris	-4°C	-2°C
Oslo	-13°C	-7°C
Rome	3°C	10°C
Warsaw	-6°C	2°C

At midnight, how many degrees colder was Paris than Rome?

degrees



End of Year Expectation

Multistep problem in context, deciding which methods to use and why.

Provide a mixture of opportunities to apply mental and formal strategies taught

At the start of June, there were 1,793 toy cars in the shop.

During June,

- 8,728 more toy cars were delivered
- 9,473 toy cars were sold.

How many toy cars were left in the shop at the end of June?

$$\begin{array}{r} 1793 \\ + 8728 \\ \hline 10521 \\ - 9473 \\ \hline 1048 \end{array}$$

Answer = 1048

Year 6 Addition Representations and Models

Compact Column Addition

Starting adding in the ones column and move up each column in order.

$$104,328 + 61,731 = 166,059$$

1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9

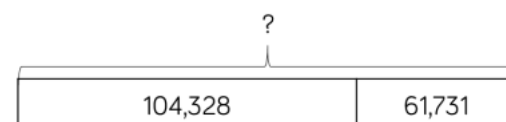
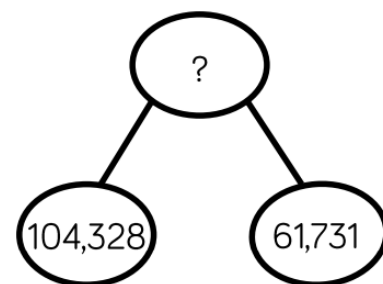
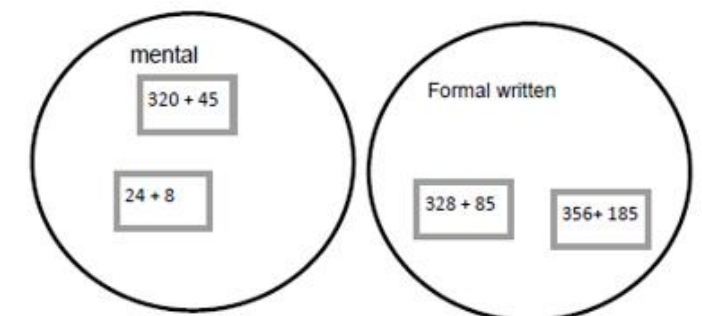
1

Representations alongside column addition

HTh	TTh	Th	H	T	O
100000	60000	4000	300	20	8
60000	10000	7000	300	20	1
100000	60000	4000	300	20	8

		5	2	2	4	7	
	+	3		5	9	0	4
		9	0		3		2

Sort these calculations. Explain which methods you would use and why.



Year 6 Subtraction

Linked to Hampshire Scheme of Learning Units 6.1, 6.7, 6.10 and 6.15

National Curriculum Objectives:

Pupils should be taught to:

- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- Use negative numbers in context, and calculate intervals across zero

The Big Ideas

Deciding which calculation method to use is supported by being able to take apart and combine numbers in many ways. For example, calculating $8 \cdot 78 + 5 \cdot 26$ might involve calculating $8 \cdot 75 + 5 \cdot 25$ and then adjusting the answer. The associative rule helps when adding three or more numbers: $367 + 275 + 525$ is probably best thought of as $367 + (275 + 525)$ rather than $(367 + 275) + 525$.

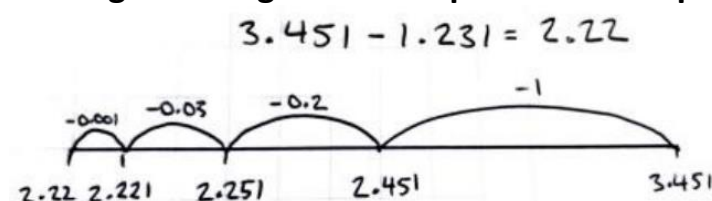
Stage 1

Column subtraction with number with more than 4-digits.

Ensure number sentences chosen would not be more suited to a more efficient mental strategy.

$$\begin{array}{r} 748128 \\ - 33294 \\ \hline 41634 \end{array}$$

Subtracting involving numbers up to 3 decimal places.



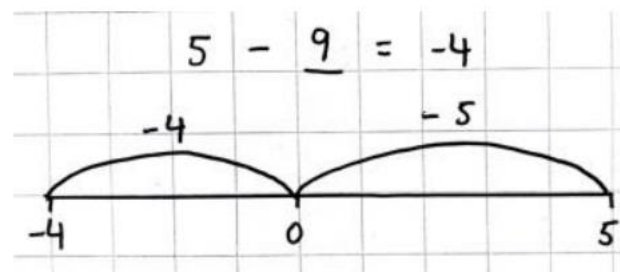
Stage 2

Subtracting with negative numbers in context.

This table shows the temperature at 9 am on three days in January.

1st January	8th January	15th January
+5°C	-4°C	+1°C

What is the difference between the temperature on 1st January and the temperature on 8th January?

 °C


End of Year Expectation

Multistep problem in context, deciding which methods to use and why.

Provide a mixture of opportunity to apply mental and formal strategies taught

One Saturday afternoon, a total of 234,869 people attended three rugby matches.

- 80,978 people attended match 1
- 72,319 people attended match 2

How many people attended match 3?

$$\begin{array}{r} 234869 \\ - 80978 \\ \hline 153891 \\ - 72319 \\ \hline 81572 \end{array}$$

match 3 = 81,572 people

Year 6 Subtraction Representations and Models

How could you change the order of the numbers in each of the calculations to make them easier to do mentally?

$$97 + 58 + 43$$

$$68 + 57 - 28$$

$$12 \times 9 \times 5$$

1 3 5 7 9 0

Use each digit card once to complete the calculation.

$$\square \square \square - \square \square \square = \square$$

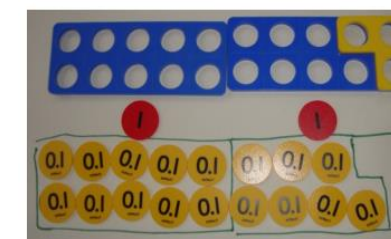
$$625 - 148 = 477$$

$$625 - 70 - \square = 477$$

$$625 - 70 - 78 = 477$$

$$2 - 0.3 = 1.7$$

$$1.7 + \square = 2$$



Children should continue to use concrete apparatus as required. They need to understand the structure of the maths.

This includes for decimals

Representations and Models – Multiplication and Division

Bar Models

?

5

5

5

5

5

5

5

5

5

5

$5 \times 5 = 25$

?

3

3

3

3

3

3

3

3

3

3

3

3

3

3

$3 \times 7 = 21$

$7 \times 3 = 21$

21

?

?

?

?

?

?

?

?

?

?

?

?

?

?

$21 \div 7 = 3$

Boys

3

3

3

3

3

Girls

3

A single bar model can be used to represent multiplication as repeated addition (for smaller numbers).

Counters, cubes or dots can be used within the bar model before using the digits within the bar model.

For division, the bar model can be used so the bar represents the whole and this can then be split into equal parts (again with smaller numbers)

It is also valuable for showing scaling problems and how 2 or more things compare to one another.

Numicon

$5 \times 4 = 20$

$4 \times 5 = 20$

$5 \times 4 = 20$

$4 \times 5 = 20$

$18 \div 3 = 6$

Numicon can be used to support multiplying and dividing of 1 digit numbers. For multiplication, it can represent repeated addition and can be used for understanding grouping for division.

When working with odd numbers, encourage children to interlock the shapes so there are no gaps. When multiplying 10s numbers can be used on top of the numbers to help find totals.

Bead Strings

$5 \times 3 = 15$

$3 \times 5 = 15$

$15 \div 3 = 5$

$5 \times 3 = 15$

$3 \times 5 = 15$

$15 \div 5 = 3$

$4 \times 5 = 20$

$5 \times 4 = 20$

$20 \div 4 = 5$

Bead strings to 100 can support children’s understanding of multiplication as repeated addition. The colour of the beads allows children to easily identify how many groups of 10 they have.

Encourage children to count in multiples as they build the numbers e.g. 4, 8, 12, 16...

When dividing, children can use the beads for grouping their whole into equal parts.

Number Tracks

$6 \times 3 = 18$

$3 \times 6 = 18$

$18 \div 3 = 6$

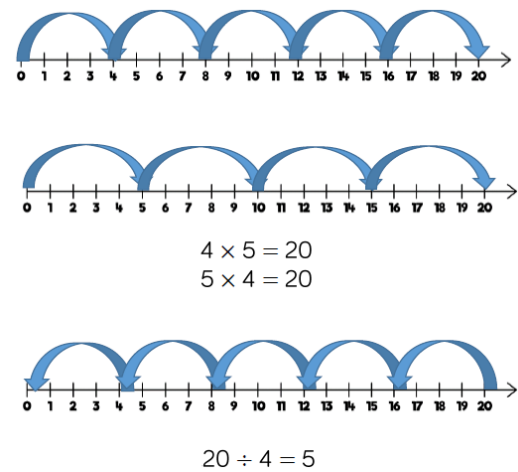
Number tracks support children in counting forwards and backwards in multiples. Use counters along the track (translucent ones will enable he children to still see the numbers).

When multiplying children place their counter on zero and then count on to find the product. For dividing, children place the ocunter on the number they are dividing and count back in equal jumps of the number they are dividing by (divisor).

This is a less efficient resource for larger number but useful for working on multiplyign and dividing by 1 digit and can support with times table knoweldge.

24

Number Lines (structured)

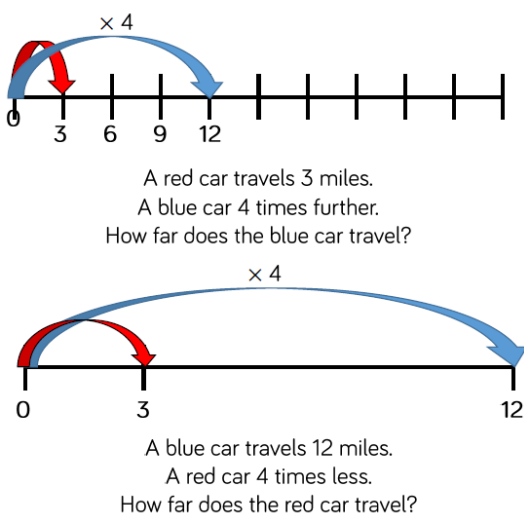


Structured number lines support children’s understanding of multiplying and dividing by one digit.

These can be used in the same way as the number tracks.

When dividing, the children need to count the number of jumps they have made in order to find their answer.

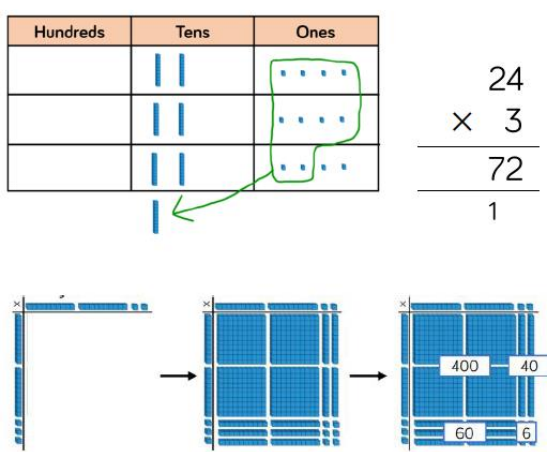
Number Line (unstructured)



Blank (unstructured) number lines can be used to represent and solve scaling problems as multiplication or division.

Number lines with marked on intervals can be used initially to represent scaling accurately.

Dienes/Base 10 (multiplication)

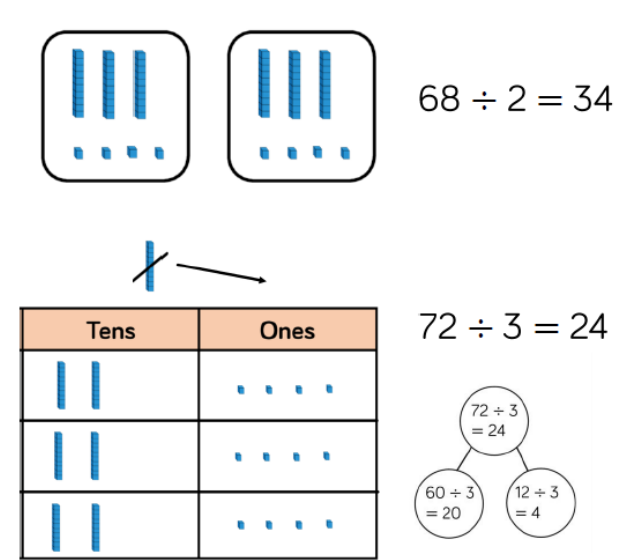


This helps children to understand column multiplication. Children should write out their calculations alongside the concrete and pictorial representations that they use.

As the numbers get larger, Dienes become less efficient but will be valuable for working with numbers up to 4 digits.

It can also be used to develop children’s conceptual understanding of the grid method for multiplying.

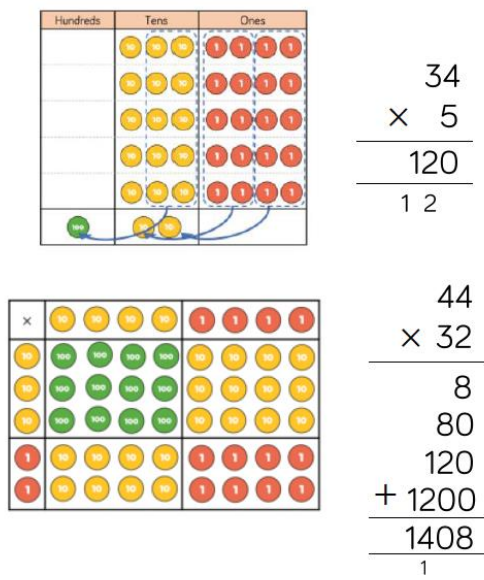
Dienes/Base 10 (division)



When numbers become larger, it can be effective to change the value of each of the Dienes resources e.g. the 1 cube becomes 10 etc.

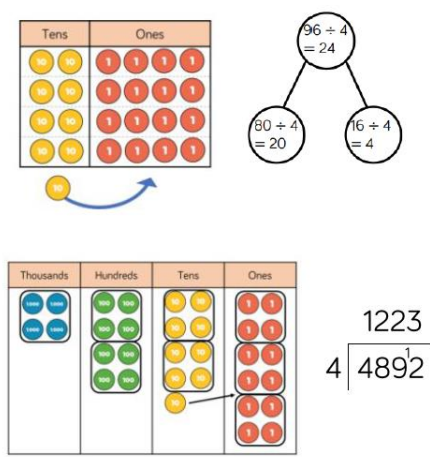
When sharing, children should begin with the larger place value and move from left to right. If there are nay left in the column they need need to exchange. Encourage children to use the part-whole model alongside their concrete and abstract calculations.

Place Value Counters (multiplication)



Using place value counters is an effective way to support the children’s understanding of column multiplication. It should be used alongside the children’s written calculations.

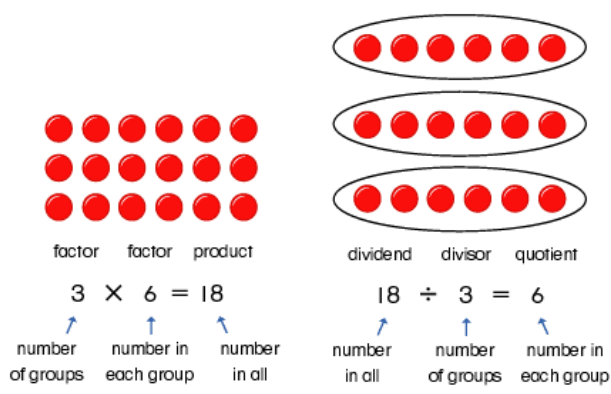
Place Value Counters (division)



Using place value counters is an effective way to support the children’s understanding of division. When working with smaller numbers, children can use place value counters to share between groups.

Place value counters can also support the children’s understanding of short division by grouping counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. The number of groups in each column represents the answer.

Arrays



Progression of Skills Multiplication and Division Overview: Years 1-6

Multiplication			
Skill	Year Group	Representations and Models	
Solve one-step problems with multiplication	1/2	Bar Model Arrays Counters	Real-life objects Number lines / tracks
Multiply 2-digit by 1-digit numbers	3	Place Value Counters Dienes	Expanded written method
Multiply 3-digit by 1-digit numbers	4	Place Value Counters Dienes	Expanded written method Short written method
Multiply 4-digit by 1-digit numbers	5	Place Value Counters Short Written Method	
Multiply 2-digit by 2-digit numbers	5	Place Value Counters Short Written Method	Expanded Written Method Grid Method
Multiply 3-digit by 2-digit numbers	5	Place Value Counters Short Written Method	Expanded Written Method Grid Method
Multiply 4-digit by 1-digit numbers	5/6	Formal Written Method	
Division			
Skill	Year Group	Representations and Models	
Solve one-step problems with division (sharing)	1/2	Bar Model Bead strings Counters	Numicon Arrays Tens frame Number lines / tracks
Solve one-step problems with division (grouping)	1/2	Real life objects Numicon Arrays	Tens Frame Counters Number lines / tracks
Divide 2-digits by 1-digit numbers (no exchange sharing)	3	Part-whole model Bar model Number line / tracks	Dienes/Base 10 Place Value Counters
Divide 2-digits by 1-digit numbers (sharing with exchange)	3	Part-whole model Bar model Number line / tracks	Dienes/Base 10 Place Value Counters
Divide 2-digits by 1-digit numbers (sharing with remainders)	3/4	Part-whole model Bar model Number line / tracks	Dienes/Base 10 Place Value Counters
Divide 2-digits by 1-digit numbers (grouping)	4/5	Place Value Counters Counters	Written Short Division Place Value Grid
Divide 3-digits by 1-digit numbers (sharing with exchange)	4	Dienes/Base 10 Bar Model	Place Value Counters Part-whole Model
Divide 3-digits by 1-digit numbers (grouping)	4/5	Place Value Counters Counters	Written Short Division Place Value Grid
Divide 4-digits by 1-digit numbers (grouping)	5	Place Value Counters	Written Short Division

		Counters	Place Value Grid
Divide multi-digits by 1-digit numbers (short division)	6	Written Short Division List of Mutliples	
Divide multi-digits by 2-digit numbers (short division)	6	Written Long Division List of Mutliples	

Year 1 Multiplication

Linked to Hampshire Scheme of Learning Units 1.3, 1.6 and 1.8

National Curriculum Objectives:

Pupils should be taught to:

- Count in multiples of twos, fives and tens.
- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

The Big Ideas:

Counting in steps of equal sizes is based on the big idea of 'unitising'; treating a group of, say, five objects as one unit of five. Working with arrays helps pupils to become aware of the commutative property of multiplication, that 2×5 is equivalent to 5×2

Stage 1

Count in multiples of 2

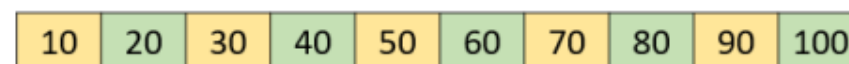
Number track



Stage 2

Count in multiples of 10

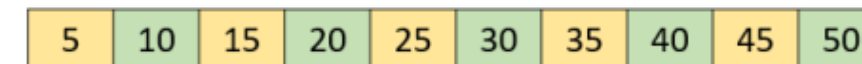
Number track



End of Year Expectation

Count in multiples of 5

Number track

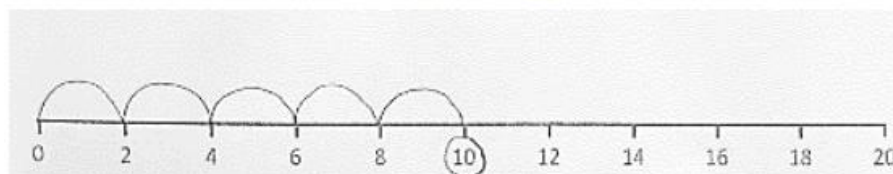
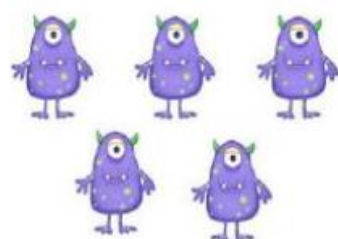


Please note that manipulatives and visual representations may be used alongside more formal recording as appropriate. It is important for pupils to explore structure and understand a concept before developing a more procedural approach, at which point all representations may be used alongside each other.

Solve one step multiplication, by calculating the answer using pictorial representations (twos).

Structured number line, e.g:

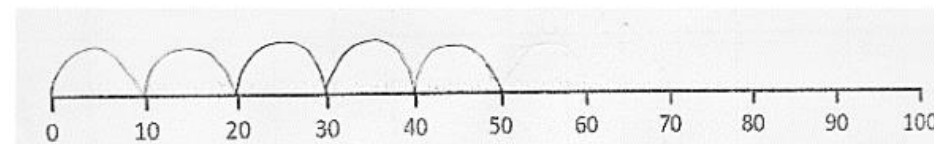
How many legs are there? Count in groups of 2.



Solve one step multiplication, by calculating the answer using pictorial representations (tens).

Structured number line, e.g:

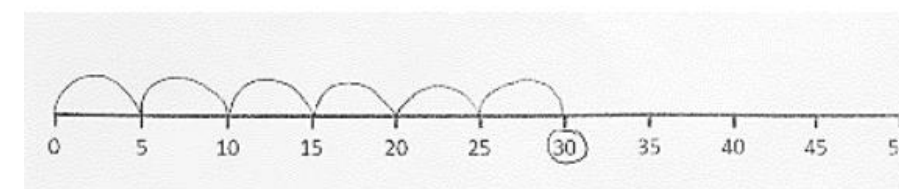
There are 10 crayons in a box.
How many crayons will I have if I buy 5 boxes?



Solve one step multiplication, by calculating the answer using pictorial representations (fives).

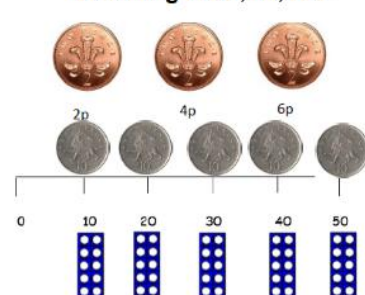
Structured number line, e.g:

Crayons come in packs of 5. How many crayons do I have?



Year 1 Multiplication Representations and Models

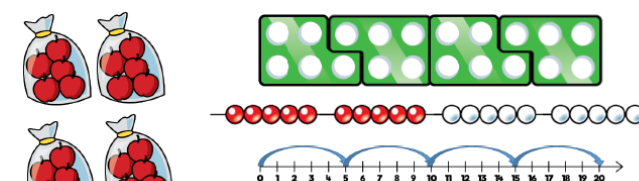
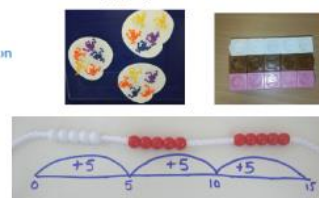
Counting in 2s, 5s, 10s



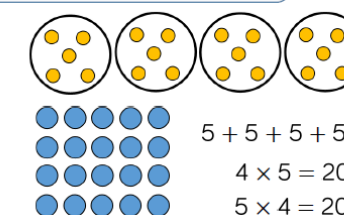
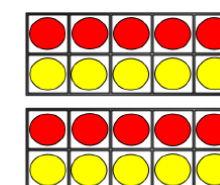
Repeated Addition

Represented with real life objects and concrete resources alongside number lines / tracks

5 frogs on each lily pad
 $5 \times 3 = 15$



One bag holds 5 apples.
How many apples do 4 bags hold?



$$5 + 5 + 5 + 5 = 20$$

$$4 \times 5 = 20$$

$$5 \times 4 = 20$$

Year 2 Multiplication

Linked to Hampshire Scheme of Learning Units 2.3, 2.6 and 2.10

National Curriculum Objectives:

Pupils should be taught to:

- Count in steps of two, three, and five from 0, and in tens from any number, forward and backward.
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in context
- Count in multiples of twos, fives and tens.
- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

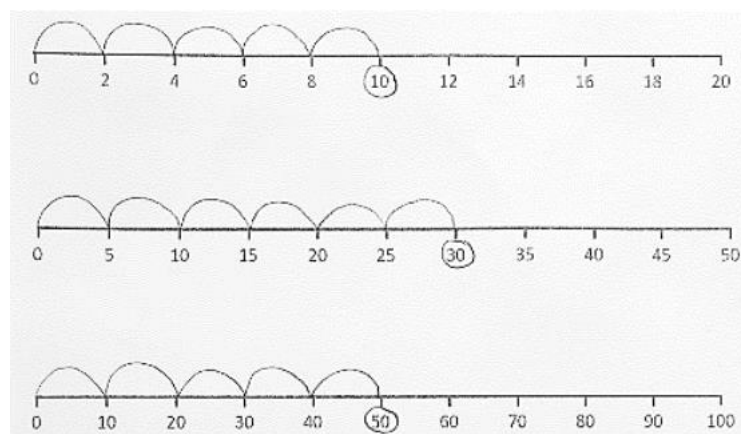
The Big Ideas:

It is important that pupils both commit multiplication facts to memory and also develop an understanding of conceptual relationships. This will aid them in using known facts to work out unknown facts and in solving problems. Pupils should look for and recognise patterns within tables and connections between them (e.g. $5 \times$ is half of $10 \times$). Pupils should recognise multiplication and division as inverse operations and use this knowledge to solve problems. They should also recognise division as both grouping and sharing.

Stage 1

Count in steps of two, five from 0 and in tens from any number, forward and backward.

Structured number line.

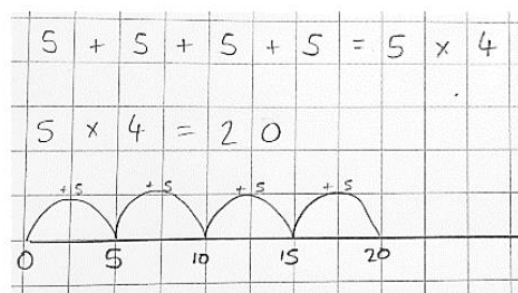


Stage 2

Solve problems involving multiplication using repeated addition.

Unstructured number line, e.g.

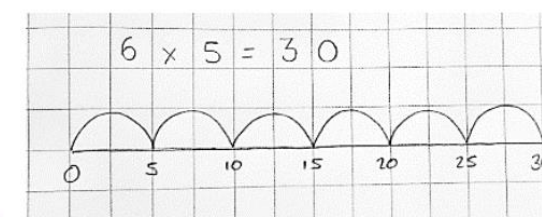
How many muffins are there altogether?



End of Year Expectation

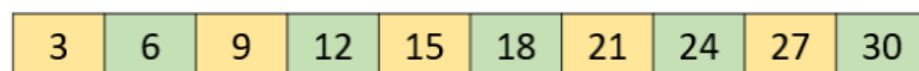
Recall and use multiplication facts for the 2, 5 and 10 multiplication tables.

Unstructured number line to 'prove it'



Count in steps of 3

Number track

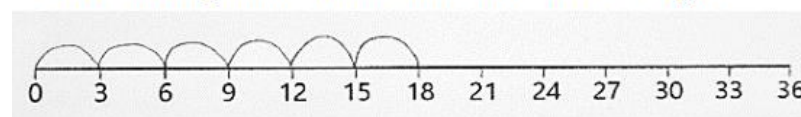


Count in steps of 3

Unstructured number line e.g.

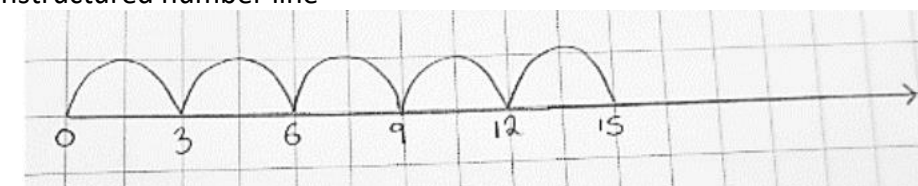
Tilly ran 3 miles every day.

How many miles has she run after 6 days?



Count in steps of 3

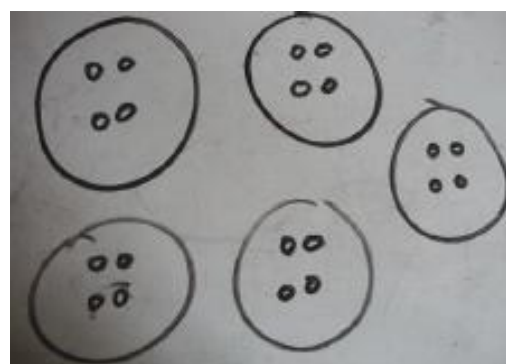
Unstructured number line



Year 2 Multiplication Representations and Models



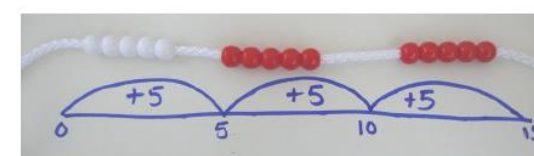
1 x 4	1 group of 4
2 x 4	2 groups of 4
3 x 4	3 groups of 4
4 x 4	4 groups of 4
5 x 4	5 groups of 4



Repeated Addition

5 frogs on each lily pad

$$5 \times 3 = 15$$



Year 3 Multiplication

Year 3 Multiplication

<p>National Curriculum Objectives:</p> <ul style="list-style-type: none">Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tablesWrite and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methodsSolve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.
--

- | |
|--|
| <p>National Curriculum Objectives:</p> <ul style="list-style-type: none">Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tablesWrite and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methodsSolve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects. |
|--|

The Big Ideas:

It is important for children not just to be able to chant their multiplication tables but also to understand what the facts in them mean, to be able to use these facts to figure out others and to use in problems. It is also important for children to be able to link facts within the tables (e.g. $5\times$ is half of $10\times$). They understand what multiplication means, see division as both grouping and sharing, and see division as the inverse of multiplication

The Big Ideas:

It is important for children not just to be able to chant their multiplication tables but also to understand what the facts in them mean, to be able to use these facts to figure out others and to use in problems. It is also important for children to be able to link facts within the tables (e.g. $5\times$ is half of $10\times$). They understand what multiplication means, see division as both grouping and sharing, and see division as the inverse of multiplication

Stage 1	
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44
45	46
47	48
49	50
51	52
53	54
55	56
57	58
59	60
61	62
63	64
65	66
67	68
69	70
71	72
73	74
75	76
77	78
79	80
81	82
83	84
85	86
87	88
89	90
91	92
93	94
95	96
97	98
99	100

Recall and use multiplication facts for the 3, 4 and 8 multiplication table.

Unstructured number line, e.g:

$$4 \times 6 = \square$$

$4 \times 6 = 4 + 4 + 4 + 4 + 4 + 4$

$4 \times 6 = 24$

Recall and use multiplication facts for the 3, 4 and 8 multiplication table.

Unstructured number line, e.g:

$$4 \times 6 = \square$$

$4 \times 6 = 4 + 4 + 4 + 4 + 4 + 4$

0 4 8 12 16 20 24

$4 \times 6 = 24$

Recall and use multiplication facts for the 3, 4 and 8 multiplication table.

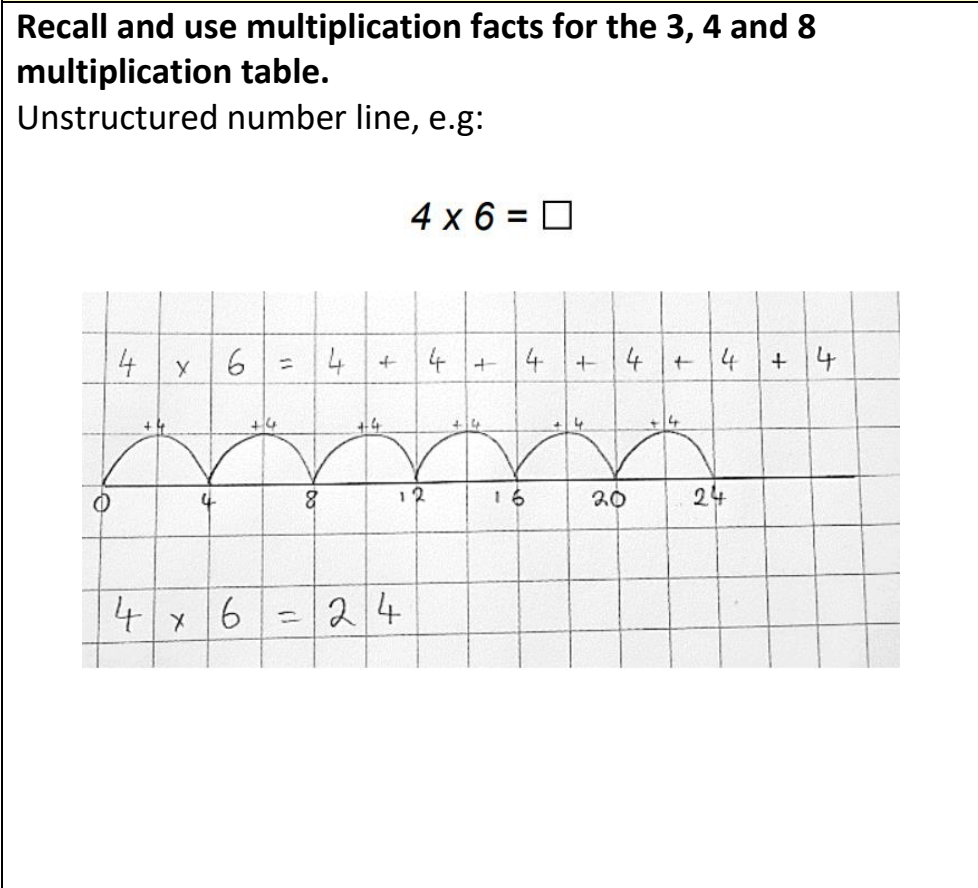
Unstructured number line, e.g:

$$4 \times 6 = \square$$

$4 \times 6 = 4 + 4 + 4 + 4 + 4 + 4$

0 4 8 12 16 20 24

$4 \times 6 = 24$



Stage 2	
1	1
2	2
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90	90
91	91
92	92
93	93
94	94
95	95
96	96
97	97
98	98
99	99
100	100

Recall and use multiplication facts for the 3, 4 and 8 multiplication tables.

Using efficient jumps on a number line when working with larger calculations, e.g:

$15 \times 4 = \square$

$15 \times 4 = 60$

Recall and use multiplication facts for the 3, 4 and 8 multiplication tables.

Using efficient jumps on a number line when working with larger calculations, e.g:

$15 \times 4 = \square$

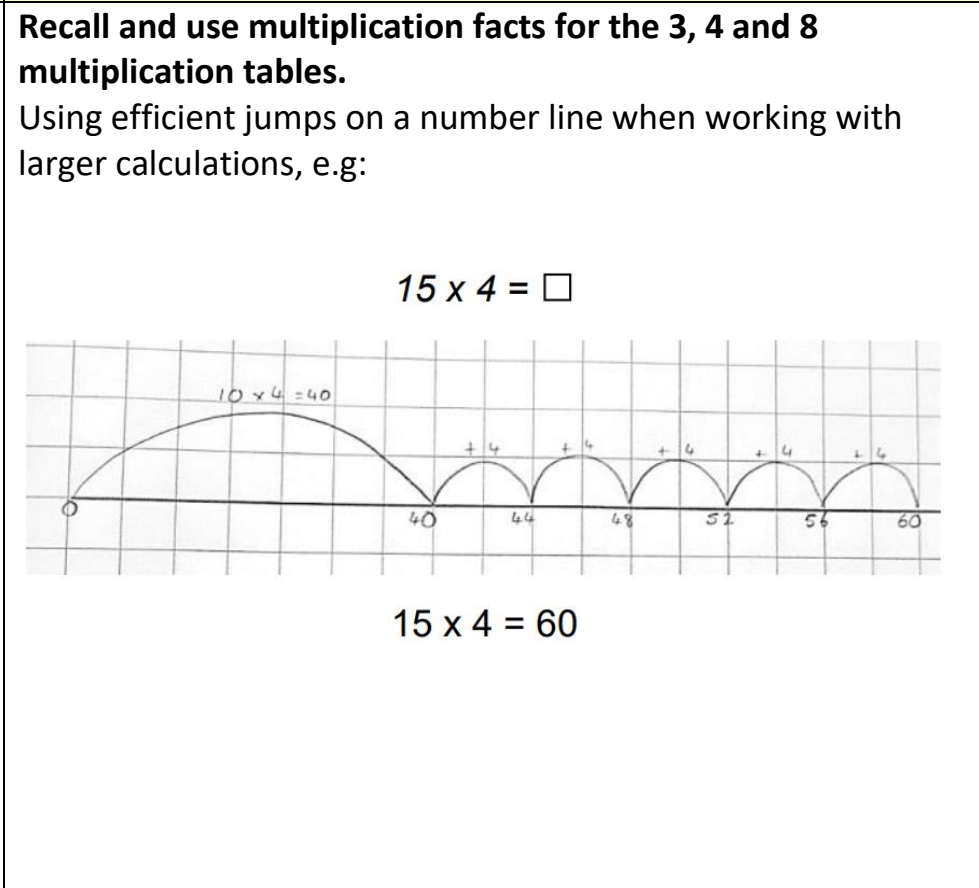
$15 \times 4 = 60$

Recall and use multiplication facts for the 3, 4 and 8 multiplication tables.

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Recall and use multiplication facts for the 3, 4 and 8 multiplication tables.

Using efficient jumps on a number line when working with larger calculations, e.g:

$15 \times 4 = \square$

$15 \times 4 = 60$

End of Year Expectation	
1	2
3	4
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13	14
15	16
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19	20
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53	54
55	56
57	58
59	60
61	62
63	64
65	66
67	68
69	70
71	72
73	74
75	76
77	78
79	80
81	82
83	84
85	86
87	88
89	90
91	92
93	94
95	96
97	98
99	100

Recall and use multiplication facts for the 3, 4 and 8 multiplication tables.

Box array, e.g:

$15 \times 4 = \square$

$10 \times 4 = 40$ $5 \times 4 = 20$

$40 + 20 = 60$

$10 \times 4 = 40$ $5 \times 4 = 20$

$40 + 20 = 60$

$15 \times 4 = 60$

Recall and use multiplication facts for the 3, 4 and 8 multiplication tables.

Box array, e.g:

$15 \times 4 = \square$

$10 \times 4 = 40$ $5 \times 4 = 20$

$40 + 20 = 60$

$15 \times 4 = 60$

Recall and use multiplication facts for the 3, 4 and 8 multiplication tables.

Box array, e.g:

$15 \times 4 = \square$

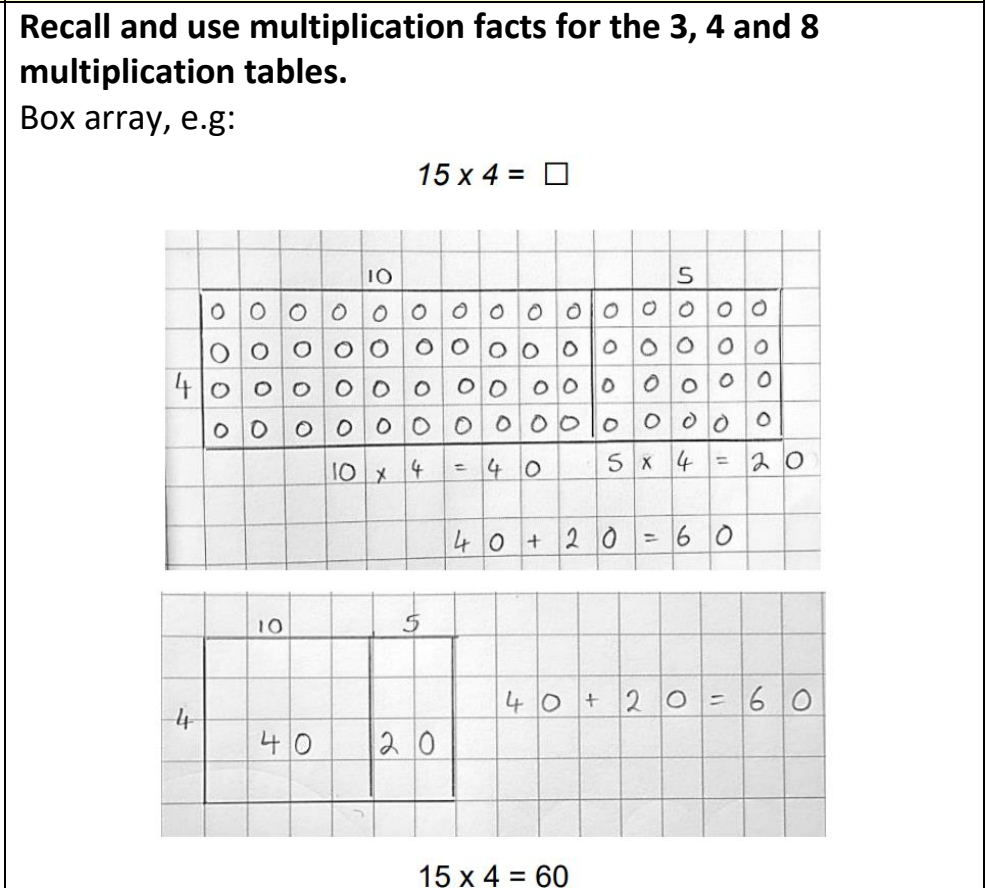
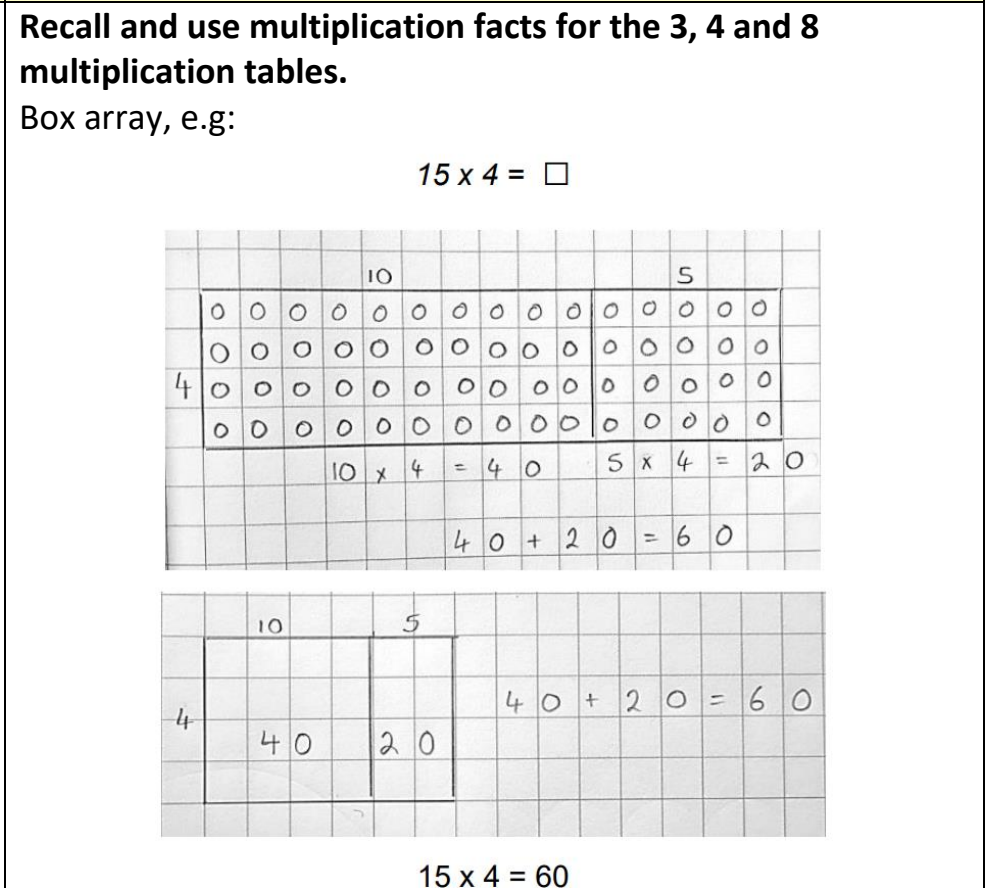
10 x 4 = 40 5 x 4 = 20

40 + 20 = 60

10 x 4 = 40 5 x 4 = 20

40 + 20 = 60

$15 \times 4 = 60$



Recall and use multiplication facts for the 3, 4 and 8 multiplication tables.

Box array, e.g:

$15 \times 4 = \square$

10 x 4 = 40 5 x 4 = 20

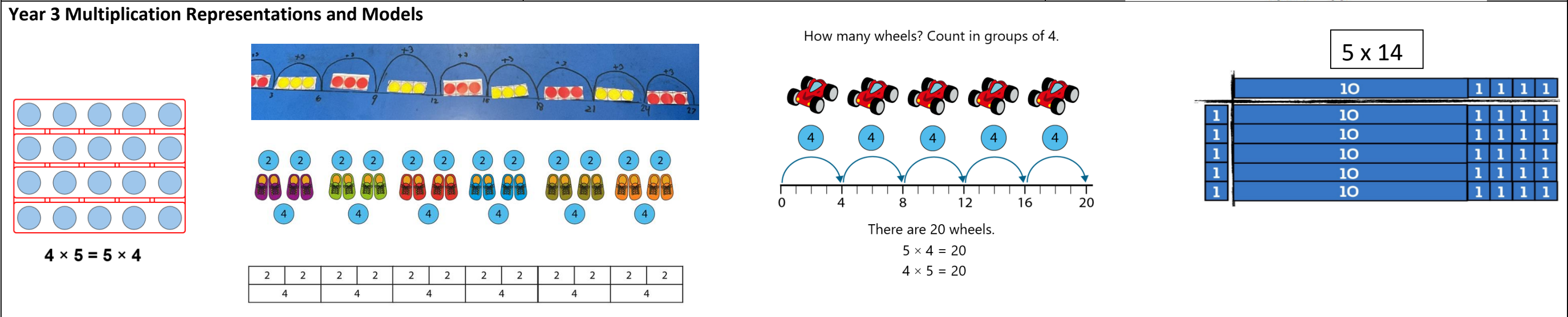
40 + 20 = 60

10 x 4 = 40 5 x 4 = 20


40 + 20 = 60

$15 \times 4 = 60$

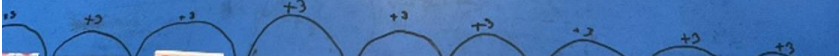
Year 3 Multiplication Representations and Models




Year 3 Multiplication Representations and Models





$4 \times 5 = 5 \times 4$





2	2	2	2	2	2	2	2	2	2	2	2
4	4	4	4	4	4	4	4	4	4	4	4

How many wheels? Count in groups of 4.

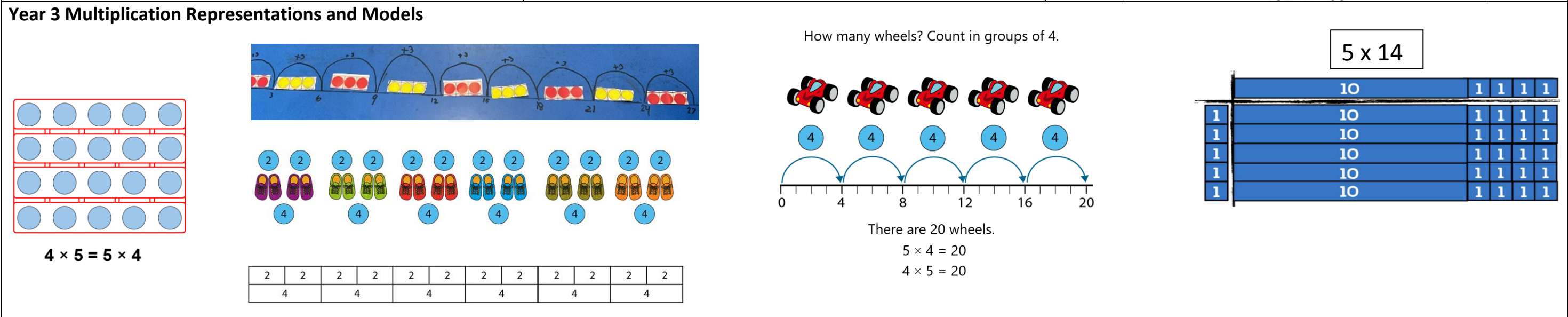
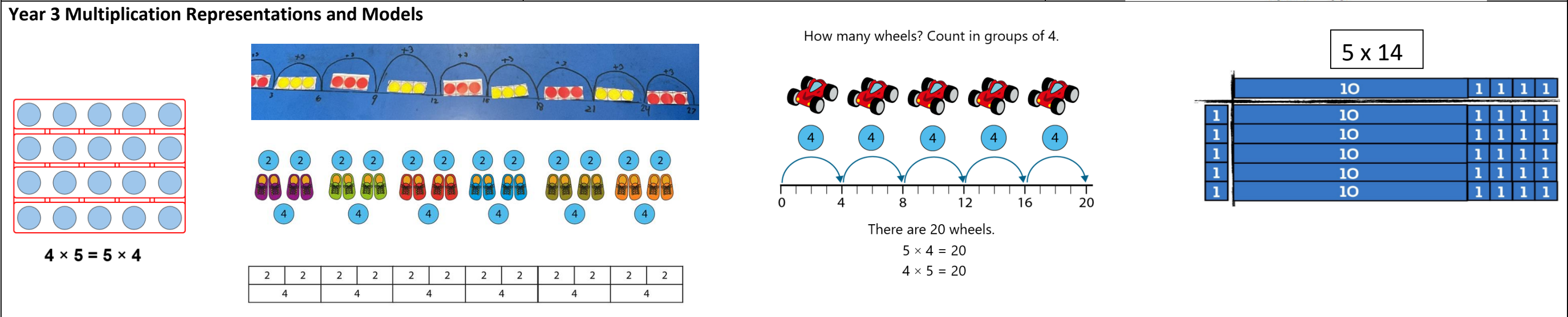
There are 20 wheels.

$5 \times 4 = 20$


$4 \times 5 = 20$

5×14

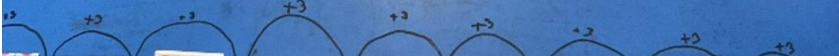
	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1




Year 3 Multiplication Representations and Models




$4 \times 5 = 5 \times 4$





2	2	2	2	2	2	2	2	2	2	2	2
4	4	4	4	4	4	4	4	4	4	4	4

How many wheels? Count in groups of 4.



There are 20 wheels.


$5 \times 4 = 20$

$4 \times 5 = 20$

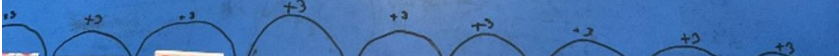
5 x 14


	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1


Year 3 Multiplication Representations and Models



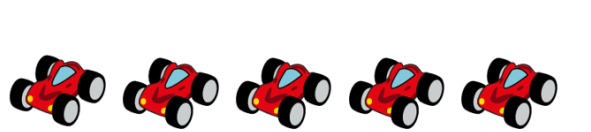
$4 \times 5 = 5 \times 4$







How many wheels? Count in groups of 4.



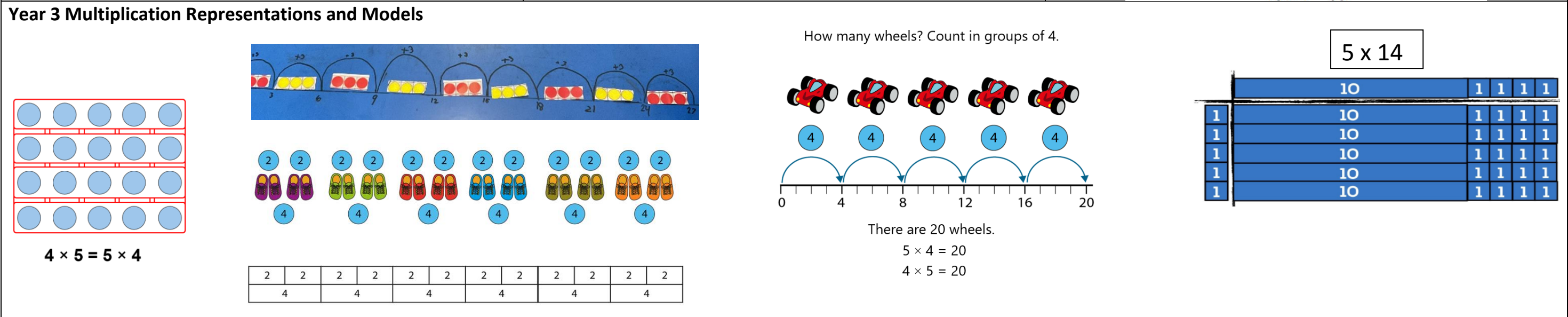
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$5 \times 4 = 20$


$4 \times 5 = 20$

5 x 14

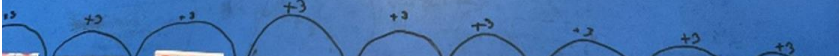
	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1




Year 3 Multiplication Representations and Models




$4 \times 5 = 5 \times 4$





2	2	2	2	2	2	2	2	2	2	2	2
4	4	4	4	4	4	4	4	4	4	4	4

How many wheels? Count in groups of 4.



There are 20 wheels.


$5 \times 4 = 20$

$4 \times 5 = 20$

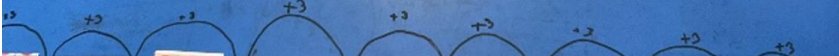
5×14


	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1

Year 3 Multiplication Representations and Models




$4 \times 5 = 5 \times 4$





2	2	2	2	2	2	2	2	2	2	2	2
4	4	4	4	4	4	4	4	4	4	4	4

How many wheels? Count in groups of 4.



There are 20 wheels.


$5 \times 4 = 20$

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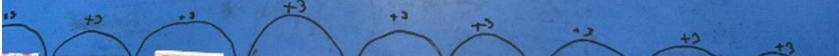
5×14


	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1

Year 3 Multiplication Representations and Models




$4 \times 5 = 5 \times 4$





2	2	2	2	2	2	2	2	2	2	2	2
4	4	4	4	4	4	4	4	4	4	4	4

How many wheels? Count in groups of 4.



There are 20 wheels.


$5 \times 4 = 20$

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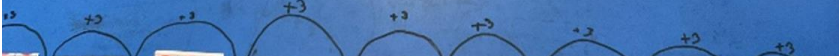
5×14


	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1

Year 3 Multiplication Representations and Models




$4 \times 5 = 5 \times 4$





2	2	2	2	2	2	2	2	2	2	2	2
4	4	4	4	4	4	4	4	4	4	4	4

How many wheels? Count in groups of 4.



There are 20 wheels.


$5 \times 4 = 20$

$4 \times 5 = 20$

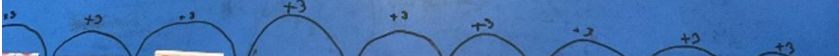
5×14


	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1

Year 3 Multiplication Representations and Models




$4 \times 5 = 5 \times 4$





2	2	2	2	2	2	2	2	2	2	2	2
4	4	4	4	4	4	4	4	4	4	4	4

How many wheels? Count in groups of 4.



There are 20 wheels.

$5 \times 4 = 20$

$4 \times 5 = 20$

5 x 14

	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1
1	10	1	1	1	1

Year 4 Multiplication

Linked to Hampshire Scheme of Learning Units 4.3, 4.9, 4.11 and 4.14

National Curriculum Objectives:

- Recall multiplication and division facts for multiplication tables up to 12×12 .
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- Recognise and use factor pairs and commutativity in mental calculations.
- Multiply 2-digit and 3-digit numbers by a 1-digit number using formal written layout.

The Big Ideas:

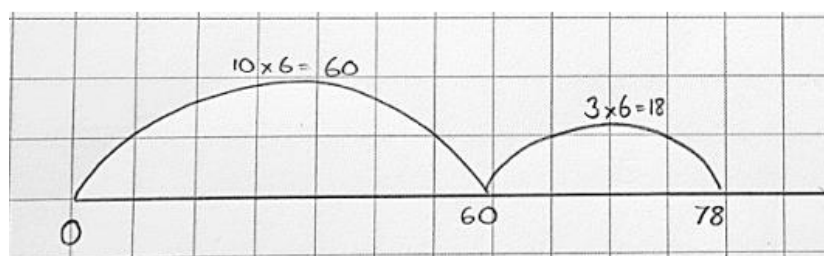
Children understand what multiplication means and see division as both grouping and sharing, and to see division as the inverse of multiplication. The distributive law can be used to partition numbers in different ways to create equivalent calculations. For example, $4 \times 27 = 4 \times (25 + 2) = (4 \times 25) + (4 \times 2) = 108$. Looking for equivalent calculations can make calculating easier. For example, 98×5 is equivalent to $98 \times 10 \div 2$ or to $(100 \times 5) - (2 \times 5)$. The array model can help show equivalences.

Stage 1

Recall multiplication and division facts for multiplication tables up to 12×12 .

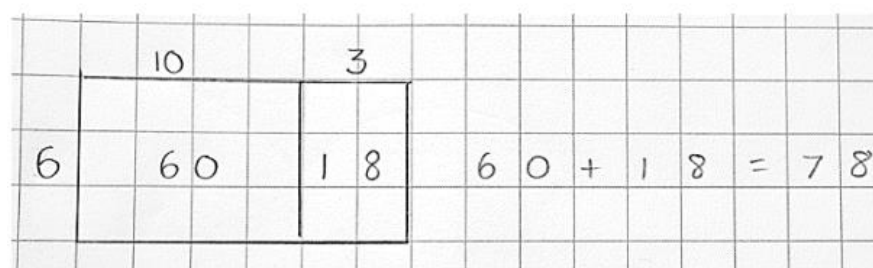
Using efficient jumps on a number line when working with larger calculations, e.g:

$$13 \times 6 = \square$$



Box Array, e.g:

$$13 \times 6 = \square$$



Stage 2

Recall multiplication and division facts for multiplication tables up to 12×12 .

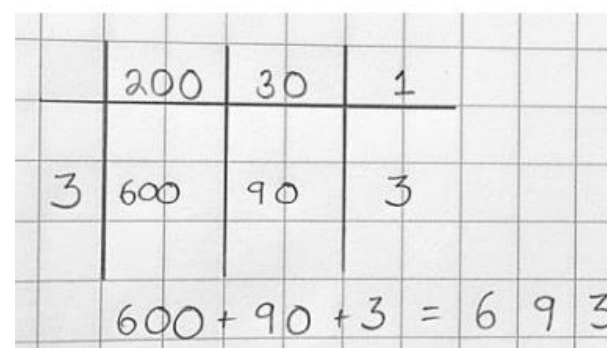
2 x 1 grid method, e.g:

$$13 \times 6 = \square$$



3 x 1 grid method, e.g:

$$231 \times 3 = \square$$

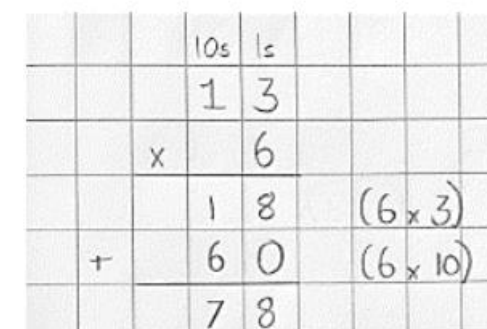


End of Year Expectation

Recall multiplication and division facts for multiplication tables up to 12×12 .

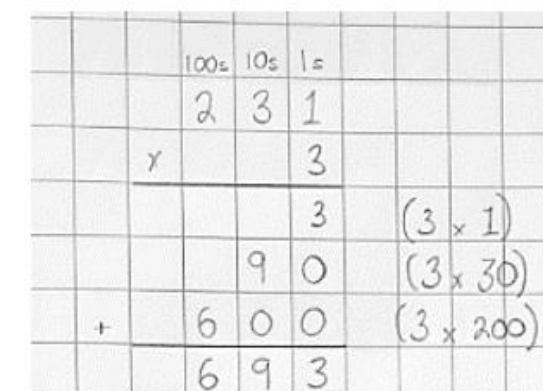
2 x 1 expanded formal method, e.g:

$$13 \times 6 = \square$$

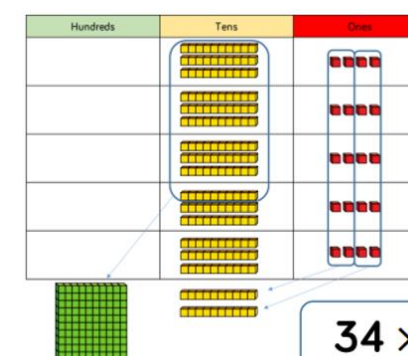
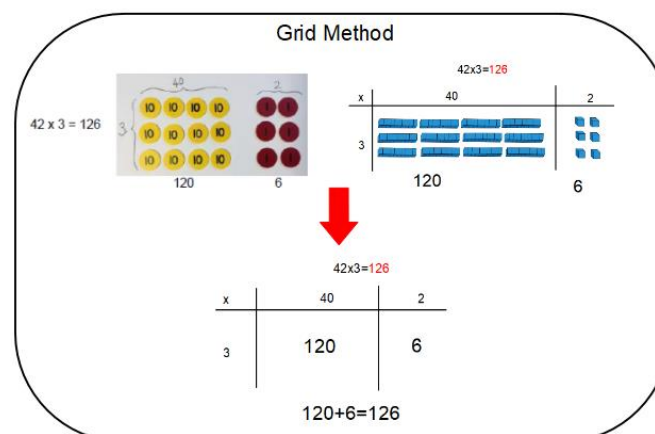
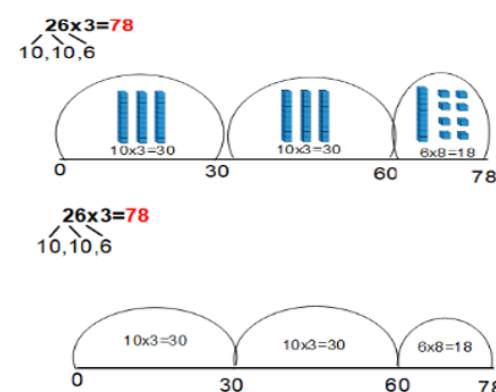


3 x 1 expanded formal method, e.g:

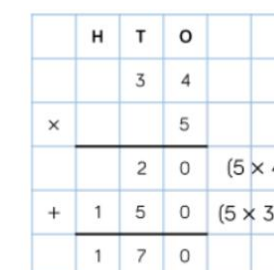
$$231 \times 3 = \square$$



Year 4 Multiplication Representations and Models



$$34 \times 5 = 170$$



Short Multiplication (UKS2)

Linked to Hampshire Scheme of Learning Units Year 5: 5.11, 5.17 and Year 6: 6.2, 6.12, 6.17

National Curriculum Objectives:

Year 5 Pupils should be taught to:

- Multiply numbers up to four digits by a 1 or 2-digit number using a formal written method, including long multiplication for 2-digit numbers.

Year 6 Pupils should be taught to:

- Multiply multi-digit numbers up to four digits by a 2-digit whole number using the formal written method of long multiplication.

The Big Ideas:

Pupils have a firm understanding of what multiplication and division mean and have a range of strategies for dealing with large numbers, including both mental and standard written methods. They see the idea of factors, multiples and prime numbers as connected and not separate ideas to learn. They recognise how to use their skills of multiplying and dividing in new problem solving situations

Stage 1

Multiply up to four digits by a 1-digit number,

2 x 1 formal method of short multiplication, e.g:

$$13 \times 6 = \square$$

	1	3
x		6
	7	8
	1	

Stage 2

Multiply up to four digits by a 1-digit number.

3 x 1 formal method of short multiplication, e.g:

$$231 \times 3 = \square$$

	2	3	1
x			3
	6	9	3

End of Year Expectation

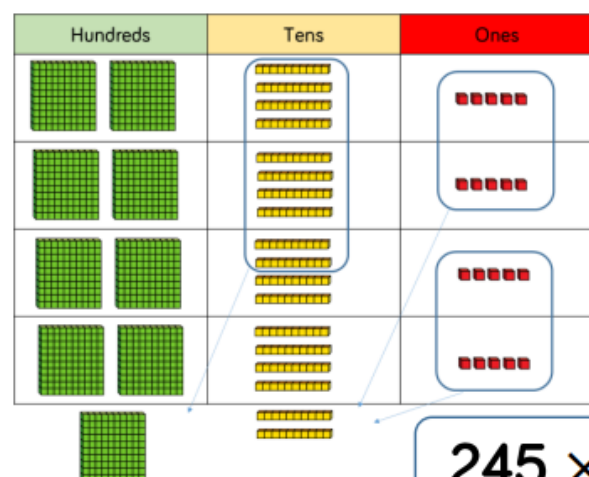
Multiply up to four digits by a 1-digit number.

4 x 1 formal method of short multiplication, e.g:

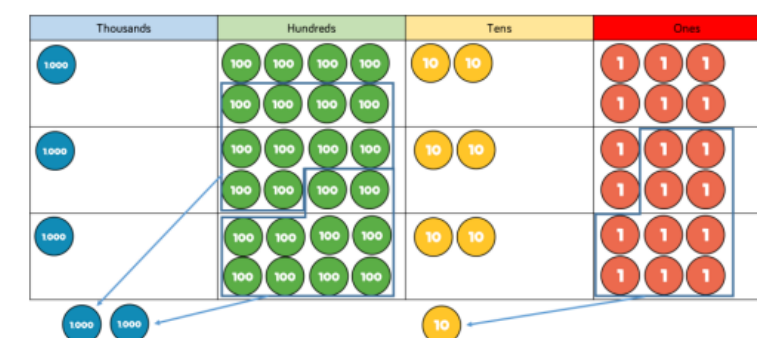
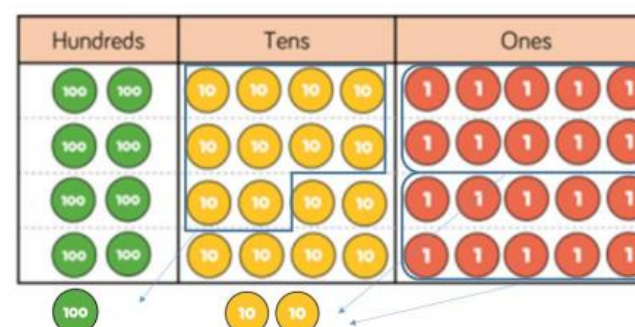
$$2731 \times 4 = \square$$

		[*] 2	[*] 7	3	1
	x				4
	1	0	9	2	4

Multiplication Representations and Models



	H	T	O
	2	4	5
x			4
	9	8	0



Long Multiplication (UKS2)

Linked to Hampshire Scheme of Learning Units Year 5: 5.11, 5.17 and Year 6: 6.2, 6.12, 6.17

National Curriculum Objectives:

Year 5 Pupils should be taught to:

- Multiply numbers up to four digits by a 1 or 2-digit number using a formal written method, including long multiplication for 2-digit numbers.

Year 6 Pupils should be taught to:

- Multiply multi-digit numbers up to four digits by a 2-digit whole number using the formal written method of long multiplication.

The Big Ideas:

Standard written algorithms use the conceptual structures of the mathematics to produce efficient methods of calculation. Standard written multiplication method involves a number of partial products. For example, 36×24 is made up of four partial products 30×20 , 30×4 , 6×20 , 6×4 .

Stage 1

Multiply up to four digits by a 2-digit number.

Formal written method of long multiplication for 2-digit numbers, e.g:

$$24 \times 16 = \square$$

		2		
		2	4	
	x	1	6	
	1	4	4	
+	2	4	0	
	3	8	4	

Stage 2

Multiply up to four digits by a 2-digit number.

Formal written method of long multiplication for 2-digit numbers, e.g:

$$124 \times 26 = \square$$

		1	2		
		1	2	4	
	x		2	6	
		7	4	4	
+	2	4	8	0	
	3	2	2	4	
	1	1			

End of Year Expectation

Multiply up to four digits by a 2-digit number.

Formal written method of long multiplication for 2-digit numbers, e.g:

$$5227 \times 43 = \square$$

				2		
			5	2	2	7
	x			4	3	
		1	5	6	8	1
+	2	0	9	0	8	0
	2	2	4	7	6	1
		1		1		

Multiplication Representations and Models

Long Multiplication 2dx2d

$$22 \times 31 = 682$$

H	T	O	
	2	2	
	3	1	x
	2	2	
6	6	0	+
6	8	2	

Begin multiplying in the ones column

$$1 \times 2 = 2$$

$$1 \times 2(20) = 2(20)$$

Then put a place holder in the ones column to show that what you are multiplying by is 10 times bigger (30 not 3)

$$3(30) \times 2 = 6(60)$$

$$3(30) \times 2(20) = 6(600)$$

	100	100	10	10	10	1	1	1	1
10	1000	1000	100	100	100	10	10	10	10
10	1000	1000	100	100	100	10	10	10	10
10	1000	1000	100	100	100	10	10	10	10
1	100	100	10	10	10	1	1	1	1
1	100	100	10	10	10	1	1	1	1

Th	H	T	O
	2	3	4
x		3	2
	4	6	8
17	10	2	0
7	4	8	8

	10	3
x		
20		
3		

Year 1 Division

Linked to Hampshire Scheme of Learning Units 1.3, 1.6 and 1.8

National Curriculum Objectives:

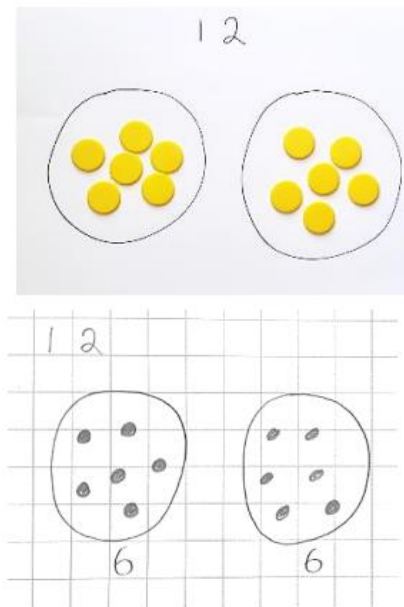
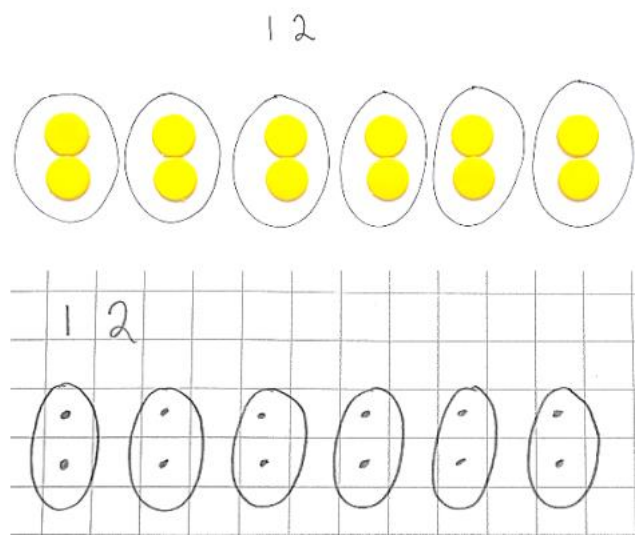
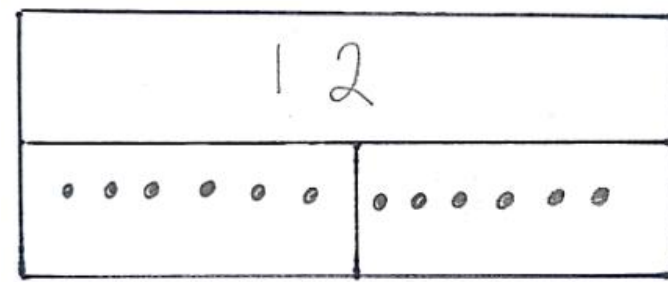
Pupils should be taught to:

- Count in multiples of twos, fives and tens.
- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

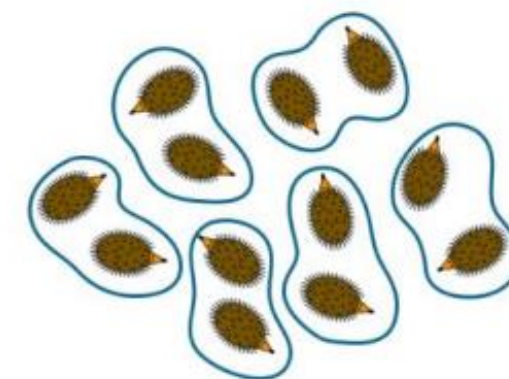
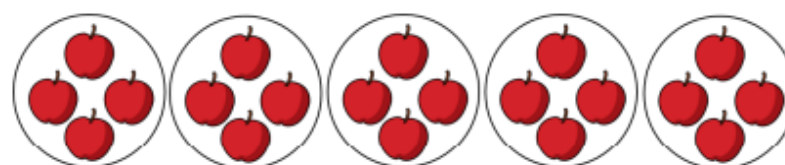
The Big Ideas:

Counting in steps of equal sizes is based on the big idea of 'unitising'; treating a group of, say, five objects as one unit of five. Working with arrays helps pupils to become aware of the commutative property of multiplication, that 2×5 is equivalent to 5×2

Please note that manipulatives and visual representations may be used alongside more formal recording as appropriate. It is important for pupils to explore structure and understand a concept before developing a more procedural approach, at which point all representations may be used alongside each other

Stage 1	Stage 2	End of Year Expectation
<p>Making equal groups – sharing. Concrete objects and pictorial representations, e.g: <i>I have 12 sweets and share them between myself and a friend (2 people), how many will we each have?</i></p>  <p>"If I share 12 equally between 2 groups, there will be 6 in each group."</p>	<p>Making equal groups – grouping. Concrete objects and pictorial representations, e.g: <i>I have 12 cookies to put in bags. If I put 2 in each bag how many bags will I need?</i></p>  <p>"There are 12 altogether. There are 6 equal groups of 2."</p>	<p>Making equal groups (including finding half of a quantity). Bar models, e.g: <i>I had 12 grapes and I ate half. How many are left?</i></p>  <p>"There are 12 altogether. They are shared into 2 equal groups. There are 6 in each group. Each group is half of the whole. I know that there are 6 grapes left."</p>

Year 1 Division Representations and Models



Year 2 Division

Linked to Hampshire Scheme of Learning Units 2.3, 2.6 and 2.10

National Curriculum Objectives:

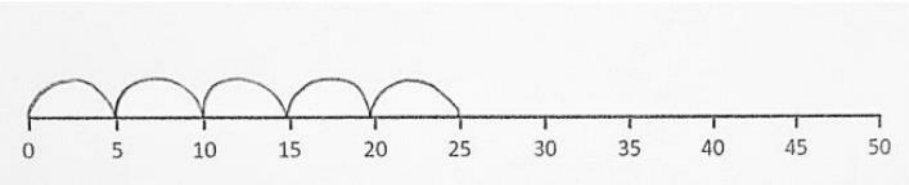
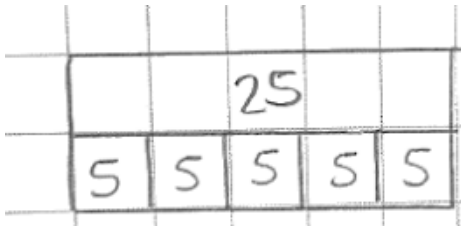
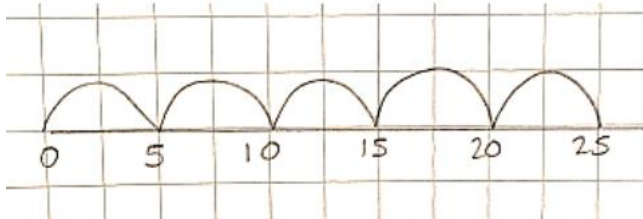
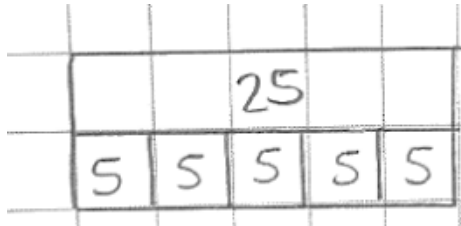
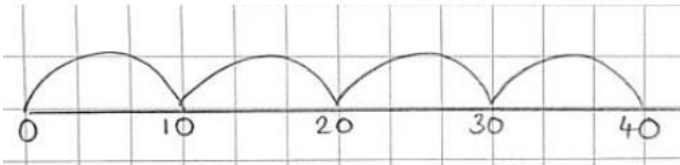
Pupils should be taught to:

- Count in steps of two, three, and five from 0, and in tens from any number, forward and backward.
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

The Big Ideas:

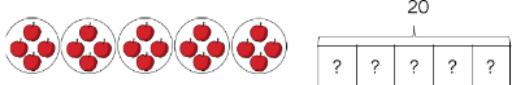
It is important that pupils both commit multiplication facts to memory and also develop an understanding of conceptual relationships. This will aid them in using known facts to work out unknown facts and in solving problems. Pupils should look for and recognise patterns within tables and connections between them (e.g. $5 \times$ is half of $10 \times$). Pupils should recognise multiplication and division as inverse operations and use this knowledge to solve problems. They should also recognise division as both grouping and sharing. The recognition of pattern in multiplication helps pupils commit facts to memory, for example doubling twice is the same as multiplying by four, or halving a multiple of ten gives you the related multiple of five

Please note that manipulatives and visual representations may be used alongside more formal recording as appropriate. It is important for pupils to explore structure and understand a concept before developing a more procedural approach, at which point all representations may be used alongside each other.


Stage 1	Stage 2	End of Year Expectation
<p>Count on in steps of two, three and five from 0. Skip counting on a structured number line, e.g:</p> $25 \div 5 = \square$  <p>Bar model representation:</p> 	<p>Count on in steps of two, three and five from 0. Skip counting on an unstructured number line, e.g:</p> $25 \div 5 = \square$  <p>Bar model representation:</p> 	<p>Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables. Number line or bar model to 'prove it'</p> <p>Ben has 40 cards. He shares them equally between 4 party bags. How many cards does he put in each bag?</p>  <p><i>"If I know that $4 \times 10 = 40$, then I know $40 \div 4 = 10$".</i></p>

Year 2 Division Representations and Models

Sharing




There are 20 apples altogether.
They are shared equally between 5 bags.
How many apples are in each bag?

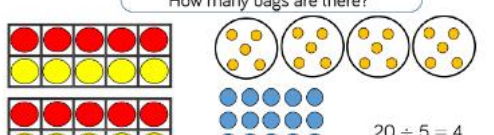


$$20 \div 5 = 4$$

Grouping

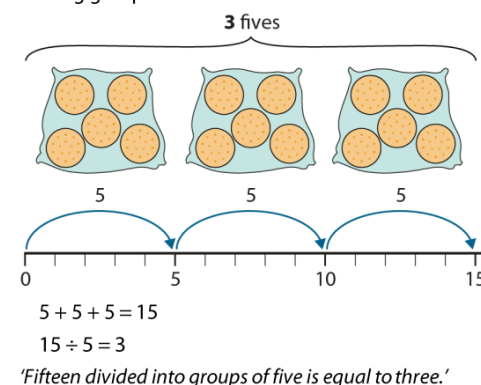


There are 20 apples altogether.
They are put in bags of 5.
How many bags are there?

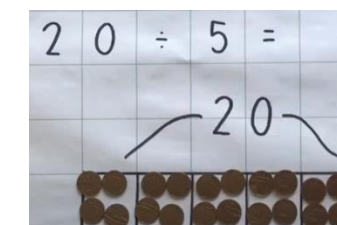
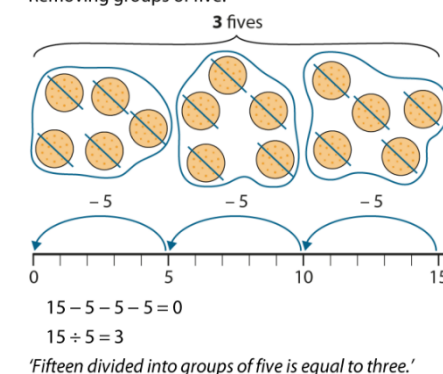


$$20 \div 5 = 4$$

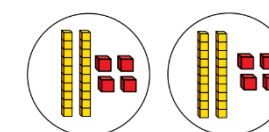
Making groups of five:



Removing groups of five:



$$48 \div 2 = 24$$



When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones.

Year 3 Division

Linked to Hampshire Scheme of Learning Units 3.3, 3.9, 3.11 and 3.14

National Curriculum Objectives:

Pupils should be taught to:

- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know

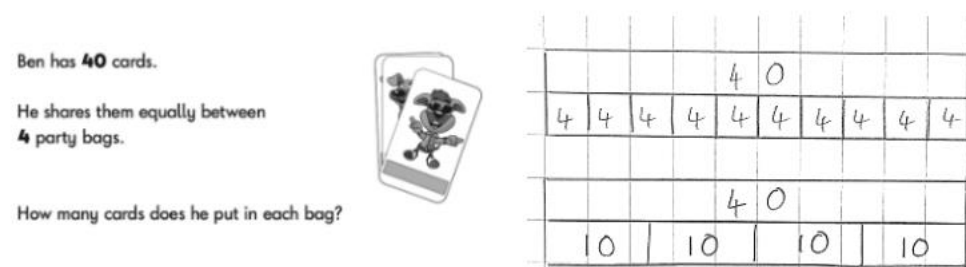
The Big Ideas:

It is important for children not just to be able to chant their multiplication tables but also to understand what the facts in them mean, to be able to use these facts to figure out others and to use in problems. It is also important for children to be able to link facts within the tables (e.g. $5 \times$ is half of $10 \times$). They understand what multiplication means, see division as both grouping and sharing, and see division as the inverse of multiplication

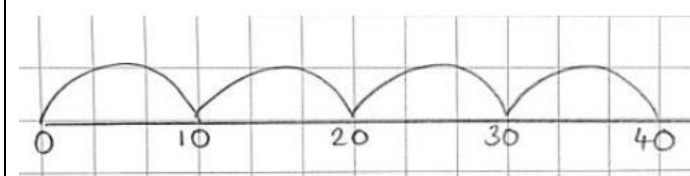
Stage 1

Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables.

Number line or bar model to 'prove it'



"If I know that $4 \times 10 = 40$, then I know $40 \div 4 = 10$ ".

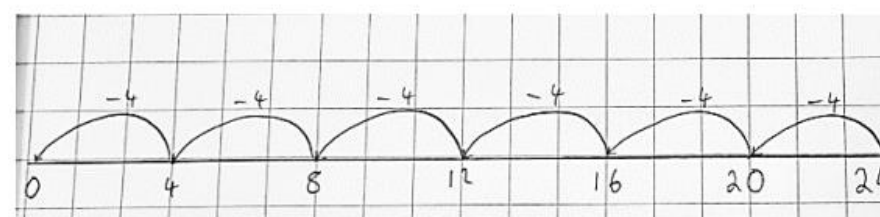


Stage 2

Recall and use multiplication and division facts for the 3, 4 and 8 multiplication.

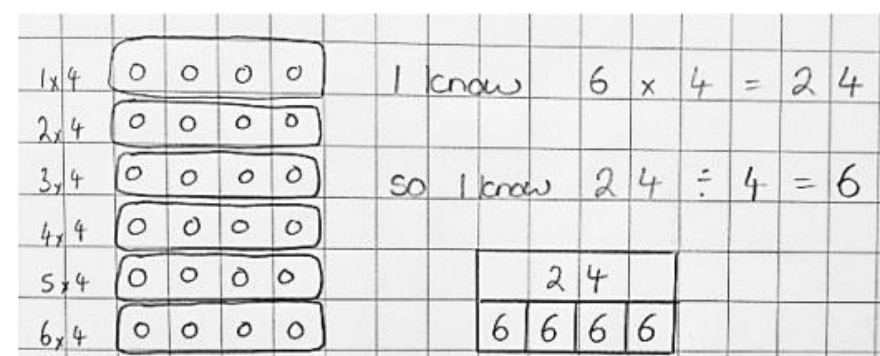
Counting back on a number line, e.g:

$$24 \div 4 = \square$$



Arrays and bar model, e.g:

$$24 \div 4 = \square$$

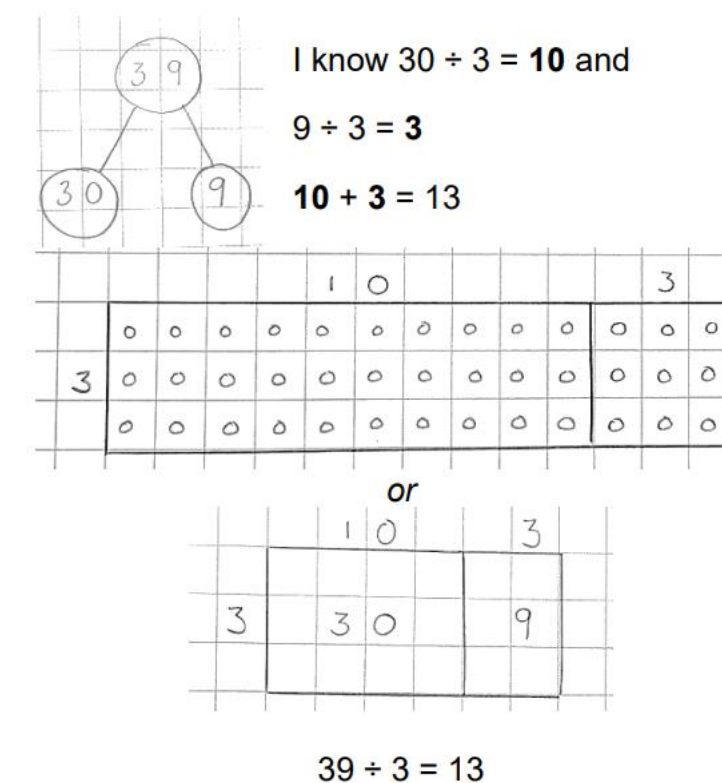


End of Year Expectation

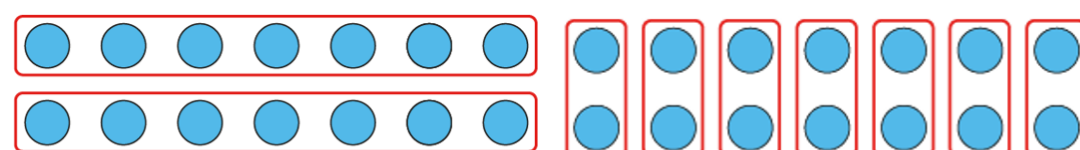
Recall and use multiplication and division facts for the 3, 4 and 8 multiplication.

Partitioning and box arrays, e.g:

39 sweets shared between 3 children. How many sweets each?

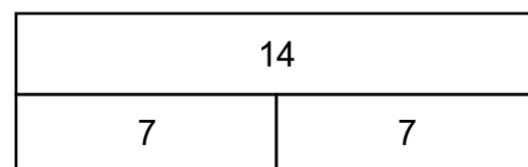


Year 3 Division Representations and Models

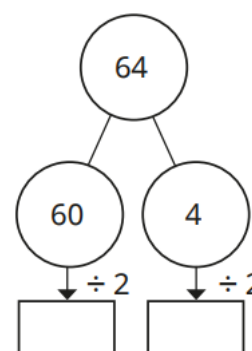


$$14 \div 2 = 7$$

$$14 \div 2 = 7$$



$$64 \div 2 = \underline{\quad}$$



Year 4 Division

Linked to Hampshire Scheme of Learning Units 4.3, 4.9, 4.11 and 4.14

National Curriculum Objectives:

Pupils should be taught to:

- Recall multiplication and division facts for multiplication tables up to 12×12 .
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- Recognise and use factor pairs and commutativity in mental calculations.
- Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers (non-statutory)

The Big Ideas:

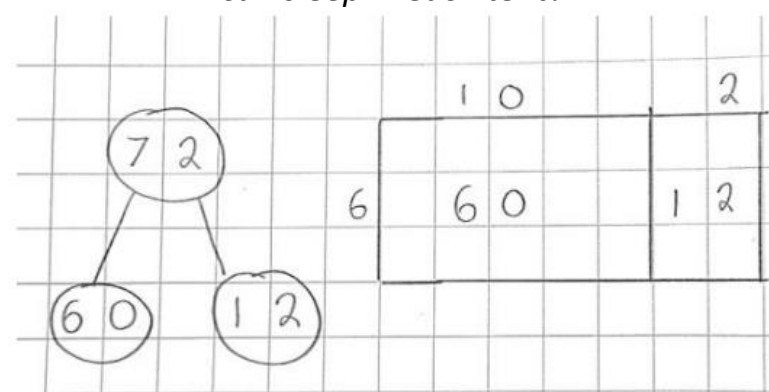
Children understand what multiplication means and see division as both grouping and sharing, and to see division as the inverse of multiplication. The distributive law can be used to partition numbers in different ways to create equivalent calculations. For example, $4 \times 27 = 4 \times (25 + 2) = (4 \times 25) + (4 \times 2) = 108$. Looking for equivalent calculations can make calculating easier. For example, 98×5 is equivalent to $98 \times 10 \div 2$ or to $(100 \times 5) - (2 \times 5)$. The array model can help show equivalences.

Stage 1

Recall and use multiplication and division facts for multiplication tables up to 12×12 .

Partitioning, e.g:

72 children go camping. There are 6 tents. How many children can sleep in each tent?



I know $60 \div 6 = 10$ and

$$12 \div 6 = 2$$

$$10 + 2 = 12$$

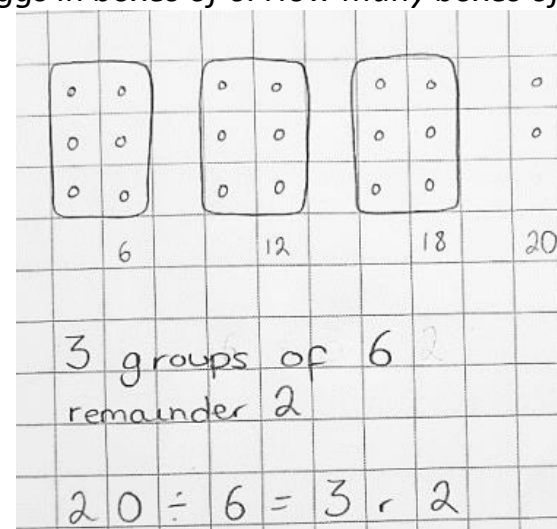
$$72 \div 6 = 12$$

Stage 2

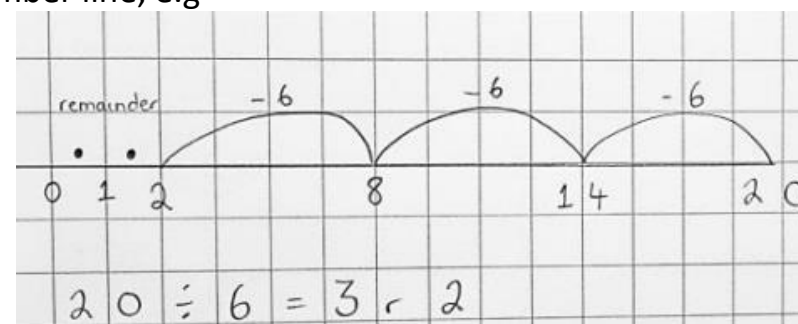
Division with remainders.

Arrays, e.g:

20 eggs in boxes of 6. How many boxes of eggs?



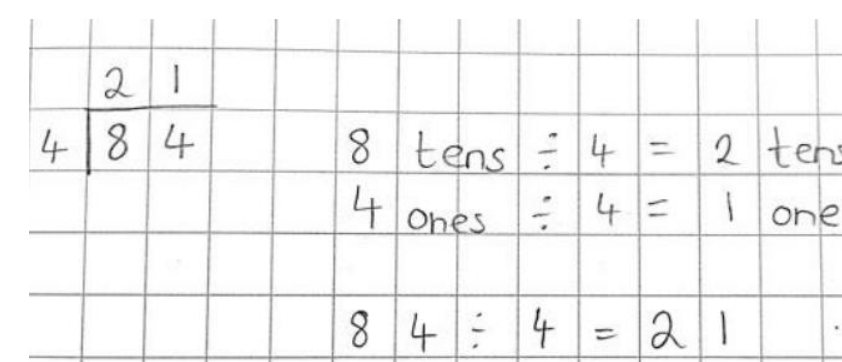
Number line, e.g



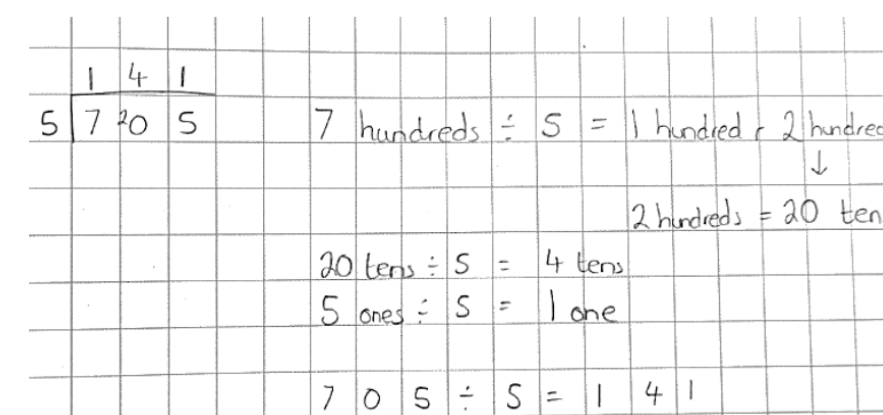
End of Year Expectation

Short division (up to 3-digit by 1-digit).

$$84 \div 4 = \square$$

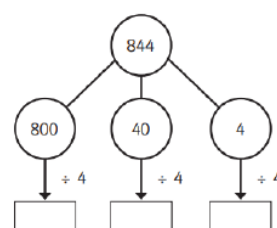
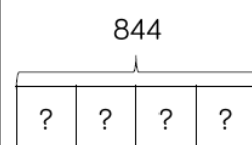


$$705 \div 5 = \square$$

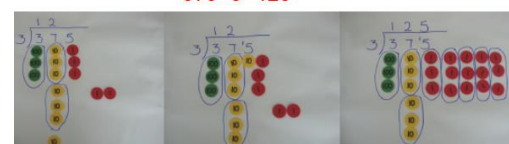


Year 4 Division Representations and Models

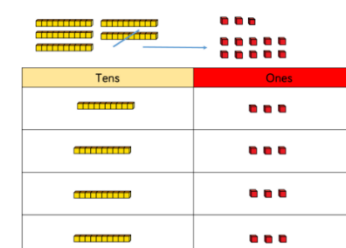
$$844 \div 4 = 122$$



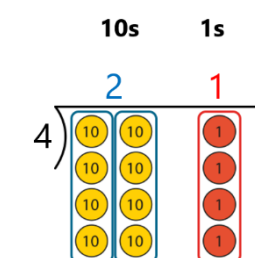
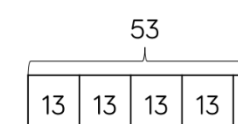
$$375 \div 3 = 125$$



Begin grouping in the hundreds column and writing how many equal group on top of the "bus stop". In the tens column there are unequal groups so you must exchange the ten that cannot be grouped into the ones column,



$$53 \div 4 = 13 \text{ r } 1$$

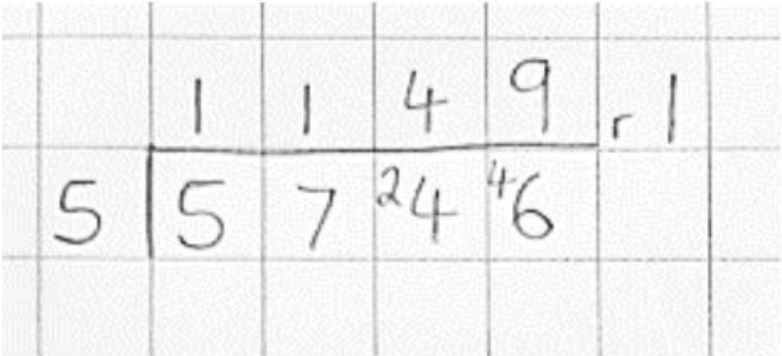
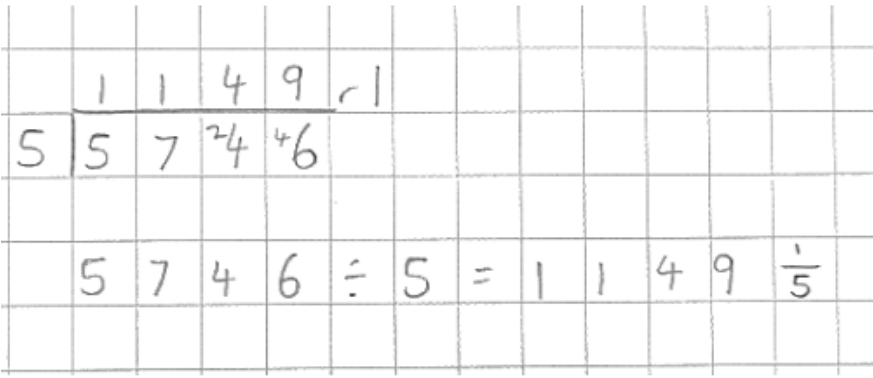
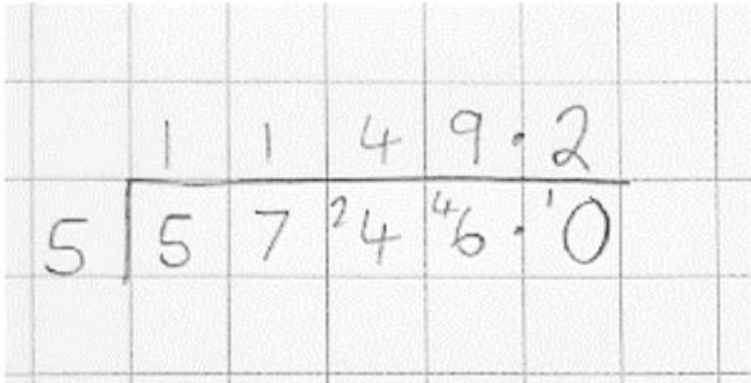


8 tens \div 4 = 2 tens
4 ones \div 4 = 1 one

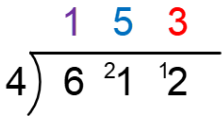
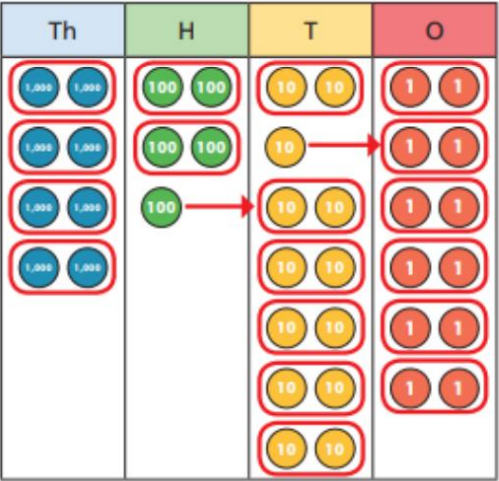
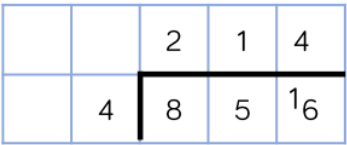
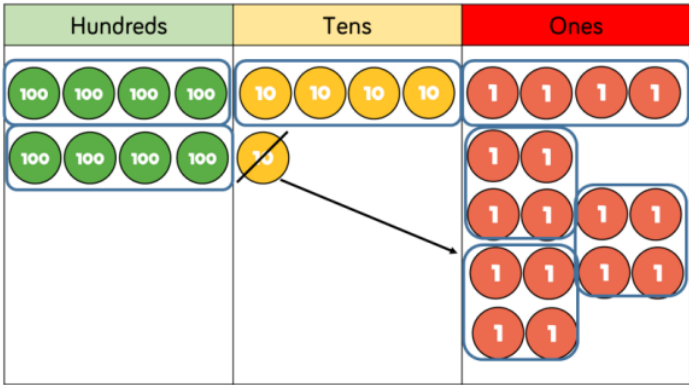
Short Division (UKS2)

Linked to Hampshire Scheme of Learning Units Year 5: 5.11, 5.17 and Year 6: 6.2, 6.12, 6.17

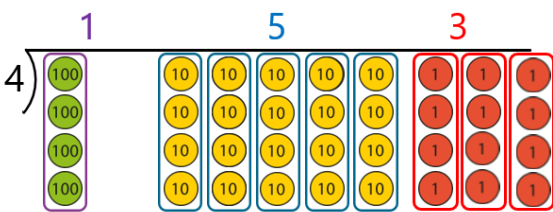
National Curriculum Objectives: Year 5 Pupils should be taught to: <ul style="list-style-type: none">Divide numbers up to four digits by a 1-digit number using the formal written method of short division and interpret remainders appropriately for the context. Year 6 Pupils should be taught to: <ul style="list-style-type: none">Divide numbers up to four digits by a 2-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context	The Big Ideas: Pupils have a firm understanding of what multiplication and division mean and have a range of strategies for dealing with large numbers, including both mental and standard written methods. They see the idea of factors, multiples and prime numbers as connected and not separate ideas to learn. They recognise how to use their skills of multiplying and dividing in new problemsolving situations.
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Stage 1	Stage 2	End of Year Expectation
Short division (up to 4-digit by 1-digit). Including remainders, e.g: $5746 \div 5 = \square$  $5746 \div 5 = 1149 \text{ r } 1$	Short division (up to 4-digit by 1-digit). Remainders as fractions, e.g: $5746 \div 5 = \square$  $5746 \div 5 = 1149 \frac{1}{5}$	Short division (up to 4-digit by 1-digit). Remainders as decimals, e.g: $5746 \div 5 = \square$  $5746 \div 5 = 1149.2$

Short Division Representations and Models



quotient
divisor)dividend



Long Division (Year 6)

Linked to Hampshire Scheme of Learning Units 6.2, 6.12 and 6.17

National Curriculum Objectives:

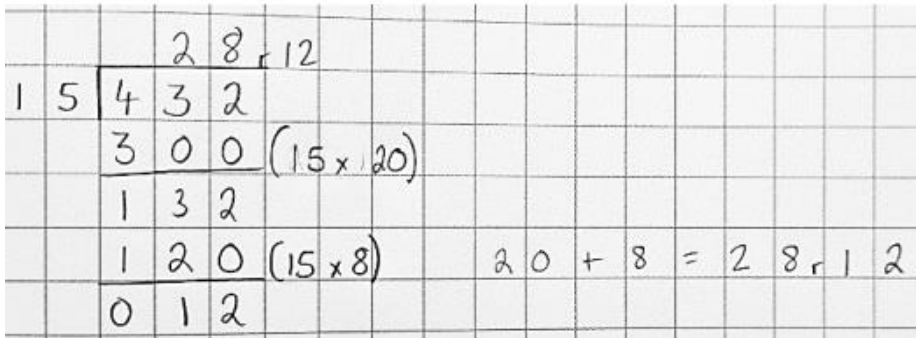
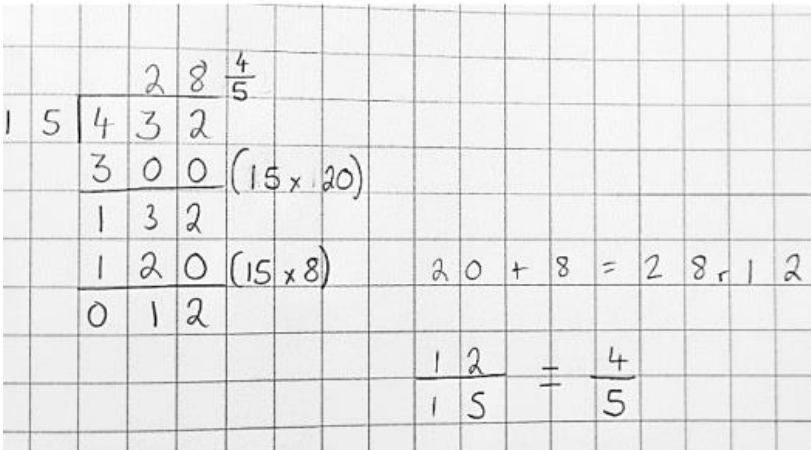
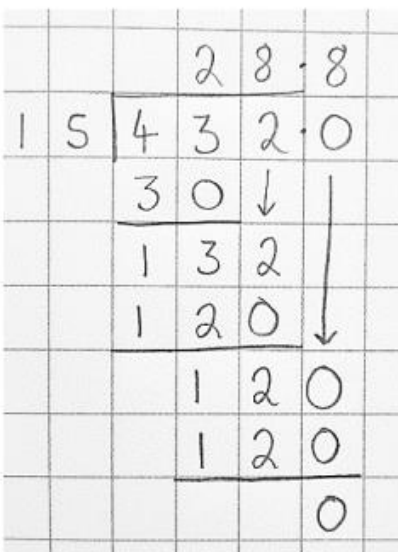
Pupils should be taught to:

- Divide numbers up to four digits by a 2-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.
- Divide numbers up to four digits by a 2-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.

The Big Ideas:

Standard written algorithms use the conceptual structures of the mathematics to produce efficient methods of calculation. Standard written multiplication method involves a number of partial products. For example, 36×24 is made up of four partial products 30×20 , 30×4 , 6×20 , 6×4 .

Please note that pupils should not move on to this method until they are conceptually and procedurally secure with strategies outlined in previous year groups. As a result, some pupils may not complete the long division strategies whilst in Key Stage 2.

Stage 1	Stage 2	End of Year Expectation
<p>Long division. Chunking method, e.g:</p> $432 \div 15 = \square$ 	<p>Long division. Chunking method with fraction remainders, e.g:</p> $432 \div 15 = \square$ 	<p>Long division. Formal written method, e.g:</p> $432 \div 15 = \square$ 

		0	3	6
1	2	4	3	2
	-	3	6	0
			7	2
	-		7	2
				0

($\times 30$)
 $12 \times 1 = 12$
 $12 \times 2 = 24$
 $12 \times 3 = 36$
 $12 \times 4 = 48$
 $12 \times 5 = 60$
 $12 \times 6 = 72$
 $12 \times 7 = 84$
 $12 \times 8 = 96$
 $12 \times 9 = 108$
 $12 \times 10 = 120$

Children can write out multiples to support their calculations with larger remainders.